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PHYSICAL MEASUREMENTS

IN THE

PROPERTIES OF MATTER

AND IN

HEAT

BY

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BERKELEY, CALIFORNIA
1910

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PREFACE

This manual is printed primarily for the use of the freshman students in the various engineering colleges of the University of California, and represents the laboratory side of a three-unit course consisting of one lecture, one recitation, and one laboratory period per week throughout the year. The course is preceded by a matriculation course in Elementary Physics, and is the first part of a two-years course in General Physics, the second part which deals with Sound, Light, and Electricity being given during the sophomore year.

The predecessors of the present writers, Professor Harold Whiting, Professor Elmer E. Hall, A. C. Alexander, G. K. Burgess, Bruce V. Hill, and A. S. King, have all played a part in the evolution of this manual. Its present form, however, is largely a revision of the manual printed in 1908 by Prof. Hall and Dr. Elston. In this revision the present writers have been guided by a desire to so simplify the work as to make possible the performance of all of the experiments, with a few exceptions, during a two-hour laboratory period. Free use has been made of published texts and manuals without specific credit being always indicated. Several experiments have been taken from the sophomore course of Professor E. R. Drew, written before the present division of subjects was made.

RALPH S. MINOR,
T. SIDNEY ELSTON.

Berkeley, Cal., August, 1910.

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LIST OF EXPERIMENTS

REFERENCES

The following list of books includes all those to which reference is made in this manual. Some of them are text-books, some laboratory manuals, and other books of tables. A few of them have been placed on the window-shelf for general reference; the others can be drawn at the desk.

Text Books—

Duff (Blakiston's): Text-Book of Physics (Second Edition.

Edser: Heat for Advanced Students.

Hastings and Beach: General Physics.

Preston: Theory of Heat (Second Edition).

Watson: Text-Book of Physics (Fourth Edition, 1903).

Laboratory Manuals.—

Terry and Jones: A Manual of Practical Physics, Vol. 1.

Millikan: Mechanics, Molecular Physics and Heat.

Watson: Text-Book of Practical Physics.

Books of Tables—

Landolt and Bernstein: Physical and Chemical Tables.

Smithsonian Institute: Physical Tables.

Whiting: Physical Tables.

PHYSICAL MEASUREMENTS.

PROPERTIES OF MATTER, AND HEAT.

This book is intended to be mainly a manual of directions. It has been the aim of the authors to make it complete enough, however, so that when used in conjunction with Blakiston's Physics, the class text-book, it will cover the minimum requirement for the year's work. It is expected that the student will elect to consult the larger reference-books (a liberal number of copies of which are available at the desk) for general notions regarding physical measurements, the discussion of results, the effect of errors in observation and methods for their complete or partial elimination.

Printed directions, regarding the method of writing-up and handing-in the record of the experiments, will be found on the folder used as a cover for the record. A list of the required experiments and the order in which they are to be performed will be posted on the laboratory bulletin-board.

1. SENSITIVE BEAM BALANCE. DENSITY OF A SOLID.

Weighing by Method of Vibrations.

In weighing with a sensitive beam balance use is made of a long pointer attached to the beam and arranged to vibrate in front of a fixed scale. The more sensitive the balance the greater the angle will be through which the pointer will vibrate for a given excess mass placed in either of the pans. The pointer will swing many times back and forth before it finally comes to rest at a definite point which marks the position of

equilibrium. Time would be wasted in waiting for it to stop, and even then the indications of the moving pointer are more trustworthy than those of one which has come to rest, because the latter may not be in its true position of equilibrium, or *rest-point,* owing to friction.

To obtain the rest-point the pointer is allowed to vibrate and the turning-points of a number of consecutive swings are read, the number being so chosen as to give an even number of turning-points on one side and an odd number on the other. A little consideration will show that under this condition the point halfway between the mean of all the left-hand and the mean of all the right-hand readings is the true rest-point. This way of getting the rest-point is known as the "method of vibrations."

To weigh a body it is necessary first to know the rest-point with the two pans empty. This is the *zero* rest-point. The body, being placed now in the left-hand pan, enough standard masses are placed in the other pan to balance it. If the rest-point now be the same as before, the *weight of the body in air* is represented by the weight of the standard masses used. If the rest-point be not the same, it is best then to determine the *sensitiveness* of the balance, that is, the number of scale-divisions through which the addition of 1 mg. to the pan will shift the rest-point. From this and the difference between the two rest-points, the weight of the body in air may be obtained by calculation.

In most sensitive beam balances a centigram *rider* is used. By properly placing the rider on the graduated scale attached to the beam, the equivalent of any desired mass from 1 to 10 mg. may be added to either pan. Final adjustments can thus be made without opening the balance case. The rider should never be moved without first lowering the balance beam.

The balance must be handled with the greatest care, since any jarring or rapid vibration of the beam may injure the

knife-edges upon which the beam rests. On this account the beam should be lowered each time before a mass is placed on the pan or removed from it, and also when the weighing is completed.

To illustrate the use of the sensitive beam balance, let it be required to find the density of a cylindrical solid.

(*a*) With the pans of the balance empty, raise the beam slowly and allow the pointer to swing over four or five scale-divisions. Take and record an even number of turning-points on one side and an odd number on the other (respectively 4 and 3, say), and from them determine the rest-point. Make two determinations in this way, and take the mean as the zero rest-point. The door to the glass case should always be closed when determining the rest-point.

(*b*) Place the hard rubber cylinder on the left-hand pan of the balance, and add masses to the other pan until the pointer does not swing off the scale when the beam is raised. In making trials for the correct mass on the right-hand pan, raise the beam only high enough to see which side has the greater mass, in order to avoid violent rocking of the beam. Use the fractional masses and the rider to bring the pointer approximately to the zero rest-point, and then determine the rest-point by the method of vibrations. To determine how much must be added to, or subtracted from, the masses on the right-hand pan to take account of the fact that the rest-point with the loaded balance does not coincide with the zero or empty-pan rest-point, by means of the rider add 5 mg. or so to either pan and determine the sensitiveness of the balance. From the sensitiveness, the difference in the rest-points, and the masses in the right-hand pan, find the exact mass which will balance the hard rubber cylinder in air.

(*c*) *Correction for Air-buoyancy.*—Unless the body whose mass is sought has the same density as the masses used to balance it, the body will be buoyed up by the air either more

or less than the masses are buoyed up, and this will introduce an error which is by no means negligible in careful measurements. *To correct for air-buoyancy*: Measure the dimensions of the cylinder with vernier calipers, and compute its volume. Calculate the volume of the standard brass masses from their marked mass values and the density of brass (8.4 gms. per cc.) Read the thermometer and barometer. From the given volumes and the density of air at the temperature and barometric pressure at the time of the experiment (see Tables), determine the correction for air-buoyancy. Calculate the density of the cylinder.

(*d*) If the arms of the balance be unequal in length, "double weighing" is necessary. In such a case the cylinder is placed in the right-hand pan and the mass found as before. The true mass is then given by

$$m = \sqrt{m_1 \, m_2},$$

where m_1 and m_2 are the values obtained by the two weighings. The proof of this by an application of the principle of moments is left to the discretion of the student. In most sensitive balances the arms are so nearly equal that double weighing is unnecessary.

2. JOLLY'S BALANCE. SPECIFIC GRAVITY.

Jolly's balance consists of a long spiral spring suspended from an upright steel frame. The lower end of the spring carries two light pans, the lower of which is always immersed to the same depth in a beaker of water standing on a small platform attached to the frame. In one form of the balance the upper end of the spring is stationary, and the lower end carrying the pans may be raised or lowered and the elongation read from a graduated mirror fixed to the frame just behind the spring. In the other form of the balance the upper end of the spring is movable and the lower end with the two pans

is kept at a fixed mark; the elongation is read from a graduated sliding scale attached to the upper end of the spring.

The use of this balance to determine specific gravity depends on the fact that the spring obeys Hooke's law closely for small elongations, that is, the elongation is proportional to the change in the stretching force. With the lower pan immersed in water we first note the "empty-pan," or zero, scale-reading. The solid, whose specific gravity is to be found, is then placed in the upper pan, elongating the spring and requiring a readjustment along the scale. The scale-reading is again noted and the elongation determined. This elongation *represents* the weight of the solid in air. The solid is now transferred from the upper pan to the lower pan and the elongation of the spring, again from the zero position, is determined. This latter elongation represents the weight of the solid in water. From these data the elongation equivalent to the weight of water displaced by the solid may be found and the specific gravity of the solid calculated.

(*a*) *Hooke's Law Tested.*— Place a standard 5 cg. mass in the upper pan and determine the elongation. Repeat with larger masses, up to 5 gm. or more. With one form of the balance the readings are taken by bringing a bead or point of wire at the lower end of the spring into coincidence with its image in the mirror scale; with the other form the readings are taken from the vernier and sliding scale after bringing the mark at the lower end of the spring on a level with the etched line on the glass tube. From the observed elongations determine if the spring obeys Hooke's law or not.

(*b*) *Solids Heavier Than Water.*—By the method outlined above, find the weights, in air and in water, of the solids furnished. Take care that the lower pan is immersed to the same depth in the water and that no air-bubbles cling to the lower pan or to the solid when immersed. From the data obtained calculate the specific gravity of each of the solids.

(*c*) *Solids Lighter Than Water.*—Find the specific gravity of a solid which floats in water. For this purpose a sinker must be used, but it may be left in the lower pan throughout the experiment. The following readings will be found necessary: first with the upper pan empty, then with the solid in the upper pan, and again with the solid tied to or wedged under the sinker in the lower pan.

(*d*) *Liquids.*—Find the specific gravity of a salt-solution by using a solid hung by a thread instead of using the lower pan. (Salt-water will corrode the nickeled surface of the pan.) It will be necessary to find the elongation of the spring equivalent to the displacement of the solid in water as well as in the salt-solution. Explain the method used.

(*e*) How would you proceed to use Jolly's balance to weigh objects as you use the beam balance?

If a bubble of air had been carried down with the solid when immersed, would the calculated specific gravity have been greater or less than it should be?

Why is it necessary to keep the lower pan immersed always to the same depth throughout a given experiment?

What additional data would you need, if required to calculate the density from the specific gravity of any of the objects used in this experiment? Explain.

3. MODEL BEAM BALANCE.

The beam balance consists of a metal beam, supported so as to be able to rotate about a central knife-edge located vertically above the center of gravity of the beam. Near the ends of this beam, pans are hung from knife-edges. The result is that, wherever the object and the standard masses may be placed in the two pans, the vertical force which keeps them in equilibrium must pass through the knife-edge above, and so the effect upon the balance is the same as if the whole weight

of the scale-pan and included load acted at some point in the knife-edge from which the pan is hung. The distance from the central knife-edge to the knife-edge at either end of the beam is called the arm of the balance or the length of the beam.

A model beam balance is a simplified beam balance used to test the relation between the sensitiveness of the balance and its dimensions and load. By "sensitiveness" is meant the facility with which the pointer of the balance can be deflected when there is a small difference between the masses suspended from the two sides of the beam. The sensitiveness of the balance depends upon the length and mass of the beam, the load in the pans, the distance between the center of gravity of the beam and the central supporting knife-edge, and upon whether the beam is straight or curved up or down. To obtain an expression showing the character of this dependence we need to apply the principle of moments.

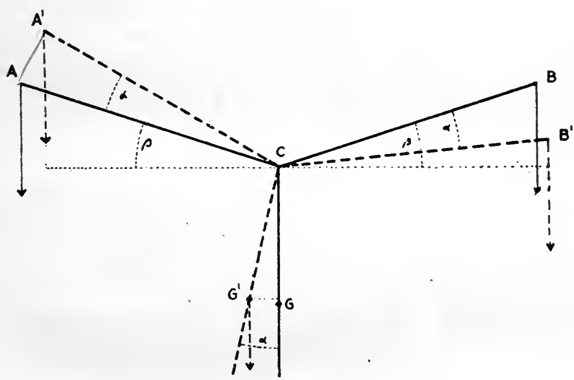


Fig. 1.

Let us suppose that in the figure the points A, C, B represent the positions of the three knife-edges and G the position of the center of gravity of the beam when the two pans are carrying equal loads; and that the points A', C, B' repre-

sent the corresponding positions of the knife-edges and G' the corresponding position of the center of gravity when a small excess mass is added to the right-hand pan.

Let m = the mass of the beam,

l = the length of the beam-arm (the two being assumed equal),

M = the mass hung on each side, including the mass of the scale-pan,

h = the distance from the central knife-edge to the center of gravity of the beam,

x = a small excess mass placed in one pan,

α = the deflection produced by the addition of x,

β = the angle, for the given load, between a horizontal line and the line drawn from the central knife-edge to the knife-edge at either end, when the beam is so placed that the two angles which can be thus formed are equal. β will be positive if the beam is concave upwards, negative if the beam is concave downwards. Applying the principle of moments for the case of equilibrium, the central knife-edge being the center of moments, we have

$$(1)\quad (M+x)\,gl\cos(\beta-\alpha) - Mgl\cos(\beta+\alpha) - mgh\sin\alpha = 0.$$

Expanding, collecting terms, and transposing,

$$[mh-(2M+x)\sin\beta]\sin\alpha = lx\cos\beta\cos\alpha, \text{ or}$$

$$(2)\qquad \frac{\tan\alpha}{x} = \frac{l\cos\beta}{mh-(2M+x)\,l\sin\beta}.$$

If the beam is straight, $\beta = 0$ and

$$(3)\qquad \frac{\tan\alpha}{x} = \frac{l}{mh}.$$

The sensitiveness is measured by the ratio, $\tan\alpha/x$. Since the expression for this ratio does not contain M, it follows that the sensitiveness in the case of a straight beam is inde-

pendent of the load; it increases with any arrangement which makes the fraction l/mh larger. In the case of a curved beam, however, it is evident from (2) that the sensitiveness is dependent upon the load and also upon the extent and direction of the curvature.

The model balance provided allows ample modification of the several quantities in equation (2), with the exception of the mass m of the beam. The following possibilities are at once apparent: (1) the length l of the beam may be varied by loosening the set screws which clamp the movable end-portions of the beam to the tubular central portion; (2) its center of gravity may be raised or lowered (thus altering h) by sliding the metal bob up or down along the pointer; (3) the load M may be increased by adding masses to the scale-pans; (4) the points of suspension of the two pans may be placed level with, above, or below the central knife-edge, thus making the beam straight, or curved up or down.

Straight Beam.

(*a*) Adjust the sliding bob so that the center of gravity of the beam lies below the central knife-edge. Change the set screws on the beams, if necessary, so as to make the two beam-arms equal in length. Place the terminal knife-edges at the zero mark, thus insuring a straight beam. Hang the two scale-pans in position. Level up the balance by means of the thumb-screws on the legs.

Place successively several 2 cg. masses in the right-hand pan, recording the deflections and noting if they are proportional to the number of masses used. Why should the deflection not be strictly proportional to the number of masses?

For convenience, select the deflection produced by the first 2 cg. mass as a measure of the sensitiveness of the balance.

(*b*) Change the length of the beam-arm. Test the sensitiveness by means of a 2 cg. mass. Compare with (*a*), and

state how the sensitiveness depends upon the length of the beam-arm, other conditions remaining unchanged; and note if the result is in agreement with the formula.

(*c*) Adjust the length of the beam-arm back to its value in (*a*), and then change the position of the center of gravity of the beam by sliding the bob up or down. Test the sensitiveness and compare with (*a*), giving your conclusions.

(*d*) Bring the bob back again to its position in (*a*). Increase the load by placing a 100 gm. mass in each scale-pan. Test the sensitiveness and compare with (*a*), giving your conclusions.

Curved Beam.

(*e*) Remove the masses from the scale-pans. Raise the terminal knife-edges to the first mark above zero, thus changing the beam from a straight beam into one which is curved up. Test the sensitiveness and compare with (*a*). Does increasing the upward curvature of the beam, other conditions remaining unchanged, increase or decrease the sensitiveness? Show that this is in agreement with the formula.

(*f*) Place a 100 gm. mass in each scale-pan. Test the sensitiveness and compare with (*e*), giving your conclusions.

(*g*) Remove the masses from the scale-pans. Lower the terminal knife-edges to the first mark below the zero, thus changing the beam into one which is curved down. Test the sensitiveness and compare with (*a*), giving your conclusions.

(*h*) Place a 100 gm. mass in each scale-pan. Test the sensitiveness and compare with (*g*), giving your conclusions.

A sensitive chemical balance is usually made with the beam curved slightly upwards when there is no load in the pans. A medium load straightens the beam and an excess load causes a downward curvature. From the results above, state how the sensitiveness of such a balance will change with the load on account of the curvature of the beam.

4. BOYLE'S LAW.

Reference.—Duff, p. 158.

The purpose of this experiment is to study the relation between the volume and the pressure of a given mass of air kept at constant temperature. According to Boyle's law the volume of a fixed mass of gas, kept at constant temperature, varies inversely as the pressure in the gas. This relation may be mathematically expressed in various ways: (1) The volumes of the gas at two different times are inversely proportional to the corresponding pressures; (2) The product of the volume and pressure at one time is equal to the corresponding product at another time, or, in other words, the product of the volume and pressure of the gas is a constant, that is,

$$p\ v = K,$$

where p and v are respectively the pressure and corresponding volume, and K is a constant whose value is fixed so long as the temperature, mass, and nature of the gas remain unchanged. This is approximately true for most of the permanent gases, provided the pressures are not very large. The higher the temperature at which the gas is held, the more closely does the law hold true.

A given mass of dry air is enclosed in an inverted, graduated glass tube which is attached to one end of a rubber tube containing mercury. At the opposite end of the rubber tube is an open glass tube. The glass tubes are clamped to vertical guides having a meter scale between them. The two clamps can be so adjusted along the guides as to vary the pressure on the enclosed air from values greater than atmospheric pressure to those which are smaller. By reading the positions of the menisci of the mercury columns in the two tubes and adding to, or subtracting from, the atmospheric

pressure as determined by the barometer, the pressure corresponding to each setting can be found. The corresponding volume of the enclosed air can be read from the graduated tube containing it.

(*a*) Clamp the tube, enclosing the air, to the vertical guide near the bottom. Raise the open tube as high along the other vertical guide as possible and clamp it to the guide. It may be necessary to pour more mercury into the open tube so as to raise the mercury level on that side. Record the temperature, the volume of the enclosed air, and the positions of the two mercury menisci. Care should be taken in reading the position of the mercury meniscus to avoid parallax. For this purpose, in sighting, stand so that the eye, the mercury surface and the image of the mercury surface formed by the mirror are in the same straight line. In reading the volume of the enclosed air, account should be taken of the curvature of the mercury meniscus. Read the barometer and record the atmospheric pressure in cm. of mercury.

(*b*) Lower the open tube 10 cm. or so at a time, repeating the readings in (*a*) for each position. Continue until the two mercury surfaces are in the same level. Record, in tabular form, the meniscus-readings, the volume v, the total pressure p, and the product $p\ v$.

(*c*) Unclamp the two tubes and raise them to points near the upper ends of the vertical guides. In doing so, care should be taken not to allow mercury to overflow. Lower the open tube 10 cm. or so at a time, repeating the readings and tabulating as in (*b*).

(*d*) On separate sheets of millimeter cross-section paper plot the following curves:

(1) With pressures as ordinates and volumes as abscissae, plot the data of (*a*), (*b*), and (*c*). Draw a smooth curve which will best represent the average position of the plotted points. What mathematical curve does it resemble? The re-

semblance would be much more marked, if the same length had been chosen to represent the pressure-unit and the volume-unit. This, however, it will not be found convenient to do.

(2) With the products $p\ v$ as ordinates and the volumes as abscissae, plot the results of (*a*), (*b*), and (*c*). Draw a smooth curve which will best represent the plotted points. What mathematical curve does it most closely resemble? If Boyle's law held strictly, what form should the curve take?

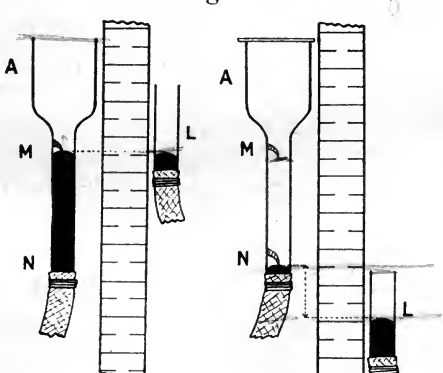
(*e*) The temperature prevailing during the experiment was room-temperature. If, instead, a higher temperature had prevailed, state with your reasons how you should expect the curves to be displaced in the plots. What effect would an increase in the mass of the air have, other conditions remaining the same as in the experiment?

What are the principal sources of error?

Determine from the two plots the volumes of the enclosed air for pressures of 50 and 100 cm. Compare.

5. THE VOLUMENOMETER.

The object of this experiment is to find the density of an irregular solid by means of the volumenometer and the balance. In the volumenometer, A is a glass tube which may be closed



122.02

79.48

42.54

70.8

Fig. 2.

at the top by a ground glass plate. It corresponds to the closed tube in the experiment on Boyle's law. As in that experiment, the pressure and volume of the air in A are varied by raising or lowering a tube containing mercury. The pressure is determined by noting the difference in the levels of the two mercury menisci, and adding to or subtracting from the atmospheric pressure as read from the barometer. The volume is unknown. The volume of a portion of the tube between two marks (M and N), however, is known.

Let the volume between M and N be k, and that above M be V. By determining the pressures when the volume of air is V (mercury meniscus at M), and again when the volume is $V+k$ (mercury meniscus at N), an equation involving Boyle's law may be written containing these two volumes and the corresponding pressures. From this equation V may be calculated. The volume of air in A may be found in this way both with and without the solid body enclosed whose volume we desire to know. The volume of this solid thus becomes known. From its volume and mass its density can be found.

(a) With the tube A uncovered bring the mercury meniscus to M, recording the pressure, evidently just equal to the atmospheric pressure. Carefully place the plate on A, so as to insure an air-tight joint. The plate must be clean and have on it only a little grease. Lower the mercury meniscus in the tube A from M to N, note the difference in level of mercury menisci in A and L, and again determine the pressure. Test for leakage by allowing the tube to remain a minute or more in this position, and make sure that the heights of the menisci do not change. Calculate k, and then, by applying Boyle's law, find V.

(b) Remove the plate, place inside the volumenometer one of the bodies whose density is to be determined, and repeat (a). From the volume V' of the air, found in this case, and

the former volume V, the volume of the body is found. Weigh the body and determine its density.

(*c*) Repeat for at least two other bodies.

(*d*) If it were possible to perfect the measuring instruments used in the course of this experiment so that they would be absolutely accurate, it would still be unreasonable to expect that the results obtained for the density would be those given in the Tables. Why?

Determine the precision of measurement of the balance, the barometer, and the volumenometer, and from these determine the precision of measurement or reliability of the final result.

What are the advantages and disadvantages of this method of determining density?

6. THE FORCE TABLE.

Reference.—Duff, pp. 36-41.

The purpose of this experiment is to determine the vector sum or resultant of two forces acting on a body in the same plane and along lines not parallel. Let the lines of direction of the two forces, f_1 and f_2, intersect in a point, O, making angles, a_1 and a_2, with an arbitrarily chosen axis, OX. The vector sum of these forces is by definition a force, f, given by the diagonal of the parallelogram formed by f_1 and f_2 as sides. Let the directional angle of f be a. By taking the projections of f_1, f_2, and f upon the perpendicular axes, OX and OY, we see by construction that

$$(1) \qquad f \sin a = f_1 \sin a_1 + f_2 \sin a_2,$$

$$(2) \qquad f \cos a = f_1 \cos a_1 + f_2 \cos a_2;$$

whence, by squaring (1) and (2) and adding,

$$(3) \qquad f^2 = f_1^2 + f_2^2 + 2f_1f_2 \cos(a_2 - a_1);$$

and, by dividing (1) by (2),

$$(4) \qquad \tan a = \frac{f_1 \sin a_1 + f_2 \sin a_2}{f_1 \cos a_1 + f_2 \cos a_2}.$$

The last two equations give the magnitude and direction of the diagonal of the parallelogram in terms of the magnitudes and directions of the sides. Equation (3) enables one to calculate the magnitude of the resultant of the two given forces, and by equation (4) the direction of this resultant can be determined.

The apparatus used to test these results consists of an adjustable iron table with circular top graduated in degrees. Pulleys can be clamped to the circumference at any chosen points. From a pin, placed in a hole in the center of the table-top, three cords pass over the pulleys and carry pans upon which known masses are placed. The masses and pans should be weighed on the platform scales.

Tested Algebraically.

(*a*) Arbitrarily take f_1, f_2, a_1, a_2 as equal respectively to $(200 + m)$ gms. wt., $(100 + m)$ gms. wt., 35°, 85°, where m is the mass of the pan holding the masses. Calculate by equations (3) and (4) the value of f and a. It will be found advisable to make these calculations and those in (*b*) before entering the laboratory to begin the experiment.

Set one of the pulleys at 35° and one at 85°. With the pin in place, put the requisite masses in the pans to make f_1 and f_2 equal to the values chosen for them. Then, if a third pulley be set 180° from the direction determined by a as calculated above, and masses corresponding to the calculated value of f be added, the three forces acting on the pin should be in equilibrium, since the third force is equal and opposite to the vector sum of f_1 and f_2. Pull out the pin and see whether the calculation is correct.

(*b*) In a similar manner calculate and test two other sets of values chosen by you.

Tested Geometrically.

(*c*) Select three new sets of values for f_1, f_2, a_1, and a_2 and proceed as follows with each set: Place a circular sheet of manila paper on the table and run the pin through it. Set the two pulleys at a_1 and a_2, and place the requisite masses on the pans. Mark with a pencil the directions of the two strings, then remove the paper and on the lines lay off distances from their intersection proportional to f_1 and f_2. Complete the parallelogram and determine from the diagonal the value of f. Now replace the paper on the table, set the third pulley opposite to f, and adjust the masses on its pan to equal the value of f as determined by the diagonal. Pull out the pin and see if the construction is correct.

(*d*) Point out the principal sources of error in the two methods used above.

If three forces acting upon a body hold it in equilibrium, how must their lines of direction intersect?

A ladder leaning against a smooth vertical wall is prevented from sliding by the reaction of the ground. What forces are acting on the ladder? Construct the line of direction of the reaction of the ground on the ladder.

7. THREE FORCES IN EQUILIBRIUM.

Reference.—Duff, p. 78.

The purpose of this experiment is to study the conditions which must be satisfied in order to produce equilibrium among three forces, two of which are mutually perpendicular. The simplest case is where the three forces are applied at the same point in the body; but the more general case where the three forces have different points of application in the body is

essentially the same, in-so-far as equilibrium is concerned, for the lines of direction of the three forces must pass through one and the same point if the forces are in equilibrium. It is quite evident that the three forces must lie in the same plane, for each of the three must be opposite to and in the same straight line with the resultant of the other two. Moreover, the resultant or vector sum of the three forces must be zero, if the forces are in equilibrium.

Let OA, OB, OC, (see Fig. 3) represent three forces f_1, f_2,

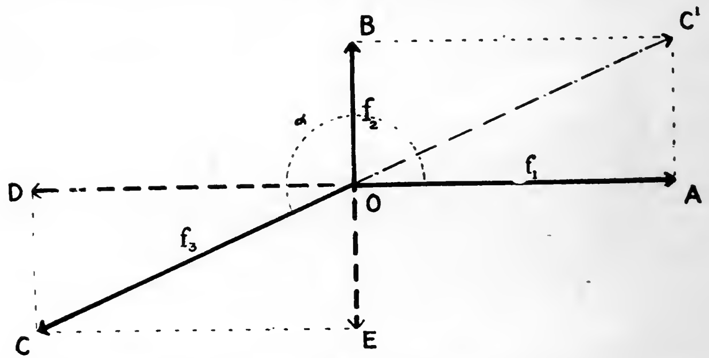


Fig. 3.

f_3, which are in equilibrium, the first two being mutually perpendicular to each other. The third force f_3, to produce equilibrium, must be equal and opposite to and in the same straight line with the resultant of the other two. Since the diagonal OC' represents the resultant of f_1 and f_2, it follows that the line OC which represents f_3 must be equal and opposite to and in the same straight line with OC'. This is the point of view considered and verified by Exp. 6.

Since the forces are in equilibrium, their resultant effect in any direction must be zero, that is, the algebraic sum of the projections of the forces in that direction must be zero. In the line DA the effect of f_1 is $f_1 \cos 0°$, the effect of f_2 is

$f_2 \cos 90°$, and the effect of f_3 is $f_3 \cos \alpha$. But the resultant effect along DA is zero, hence

$$f_1 \cos 0° + f_2 \cos 90° + f_3 \cos \alpha = 0.$$

Since $\cos 0° = 1$ and $\cos 90° = 0$, the equation becomes

$$(1) \quad f_1 + f_3 \cos \alpha = 0.$$

Similarly, the resultant effect along EB must be zero, and hence the sum of the projections of the forces into that line must be zero, or

$$f_1 \sin 0° + f_2 \sin 90° + f_3 \sin \alpha = 0.$$

Since $\sin 0° = 0$ and $\sin 90° = 1$, this equation becomes

$$(2) \quad f_2 + f_3 \sin \alpha = 0.$$

(It should be noted that $\sin \alpha$ and $\cos \alpha$ are negative for an angle in the third quadrant.) Interpreted geometrically, equation (1) shows that OA and OD are equal and opposite; equation (2) shows that OB and OE are equal and opposite.

The present experiment is intended to verify the relations (1) and (2). The apparatus consists essentially of a spring balance P (see Fig. 4), a compression spring balance Q, and

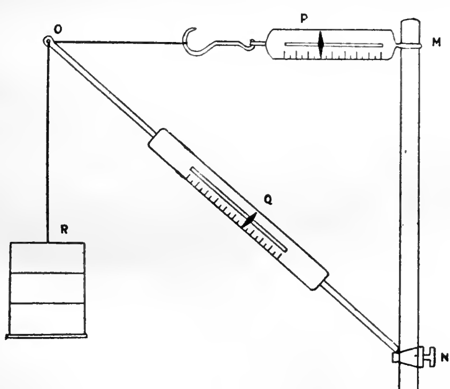


Fig. 4.

a weight-hanger *R*. The two balances are attached to a vertical rod along which they can be adjusted. The balance *P* is kept in a horizontal position so as to be constantly at right angles to the vertical force exerted by the weight-hanger. The three forces acting at *O* are respectively a horizontal pull toward the right by the balance *P*, an oblique thrust upward to the left by the rod belonging to the balance *Q*, and a vertical pull downward due to the weight of the hanger *R*. A different arrangement of forces is obtained by changing the number of masses carried by R, and by changing the direction of Q. On a sheet of paper fastened behind the balance the lines of direction of the three forces can be traced.

(*a*) Arrange the balances and the weight-hanger in the manner indicated in the figure. Hang a mass of 9 kg. on the hanger, and adjust the balance P along the rod until the angle *MOR*, as tested by means of a square, is a right angle. Support the weight of the two balances by means of the hands; note if the balance-readings are materially changed, and if they are, ask for assistance in correcting for the same. Record the readings of the two balances and the total weight of the hanger and masses. On a sheet of paper, held behind the balances, make a trace of the lines of direction of the three forces and determine the angle NOR. The sine and cosine of this angle may be found from the Tables, or directly from a measurement of the distances *OM* and *MN*.

(*b*) Repeat (*a*) twice with different masses on the hanger. Readjust the balance *P* each time so that the angle MOR shall equal a right angle.

(*c*) Change the length of the cord or chain which connects *O* with the balance *P*, and thus alter the angle NOR. Repeat the adjustments and measurements of (*a*).

(*d*) From the results in (*a*), (*b*), and (*c*) determine if the condition of equilibrium is satisfied, first by substituting the recorded values in the equations (1) and (2), and

again by constructing the triangle of forces in each case and noting if it is closed or not.

(*e*) On one of the traces draw through O a line which does not coincide with any one of the three forces. Determine the effect which each one of the three has in this line and see if the resultant effect is zero, employing the method already outlined in deriving equations (1) and (2).

What are the principal sources of error in this experiment?

How can equations (1) and (2) be used to calculate the magnitude and direction of the resultant of forces f_1 and f_2?

What is the form of the equations which would connect f_1, f_2, and f_3 if the angle NOR were not a right angle (see Exp. 6)?

8. DENSITY OF AIR.

Let a glass bulb of volume V be weighed full of air at atmospheric pressure P_1, and let M be the mass necessary to balance it. Then let the air be pumped out until the pressure is P_2, the mass as determined by weighing now being $(M-m)$, where m is the mass of air that has been pumped out between the weighings. Then if d_1 and d_2 be the densities corresponding to the pressures P_1 and P_2, it follows, from the definition of density and the interpretation of m, that

$$(1) \qquad Vd_1 - Vd_2 = m.$$

The reciprocal of the density is the volume of unit mass; hence, if Boyle's law is applied to unit mass of the air, the temperature being assumed constant, we have (see Exp. 4),

$$(2) \qquad P_1 \frac{1}{d_1} = P_2 \frac{1}{d_2}.$$

Eliminating d_2 from (1) and (2), we get

$$(3) \qquad d_1 = \frac{mP_1}{V(P_1 - P_2)}.$$

This last equation may be employed as a formula *to find the density of the air from a knowledge of the volume of the flask, the pressure before and after exhaustion, and the mass of air pumped out.* In the application it is essential that the temperature should be the same during the two weighings. (Why?) This condition is approximately satisfied in practice. If the temperature were not the same, the observed pressure in the second case would need to be corrected (through the application of Charles' law) so as to give the pressure that would have existed had the temperature been the same as during the first weighing. The volume V is obtained by weighing the bulb when empty and then when full of water at a known temperature.

(*a*) Carefully dry the flask by exhausting it several times and admitting air each time through a calcium-chloride drying-tube. Ask an assistant for instructions in regard to manipulating the pump. If moisture is visible inside the flask, it may be necessary to put in a little alcohol, rinse the flask, vaporize the alcohol over a Bunsen burner, and rinse with dry air as before. With the dried flask in connection with the drying tube, admit air at atmospheric pressure. Close the stop-cock and carefully weigh the flask. Note the temperature. Read the barometer for the pressure.

(*b*) Pump the air out until as low a pressure as possible is obtained and weigh again at this reduced pressure. Again note the temperature and record the pressure.

(*c*) Fill the flask completely with water up to the stop-cock, taking care to have no water above it. Ask an assistant to show you how to fill it. The temperature of the water should be recorded and its density found from a book of Tables. Dry the outside of the flask and then weigh. Calculate the volume of the flask.

(*d*) Using the results obtained in (*a*), (*b*), and (*c*), find the density of the air, in grams per cc., at the given tempera-

ture and atmospheric pressure. From this result the density or dry air under standard conditions (that is, at 0°C and 760 mm. pressure), may be found through the application of Boyle's and Charles' laws, or a combination of the two. If P_1, d_1 and T_1 represent the pressure, density and absolute temperature of a given kind of gas at one time, and P_2, d_2 and T_2 represent the corresponding values at another time, and so on, then it follows from a combination of the two laws that

$$(3) \qquad \frac{P_1}{d_1 T_1} = \frac{P_2}{d_2 T_2}$$

for the given kind of gas to the degree of approximation with which it observes the given laws. Making use of this relation, calculate the density of dry air under standard conditions of temperature and pressure, and compare with the value given in the Tables. Point out the chief sources of error and any other reasons for the discrepancy in the results.

9. RELATIVE DENSITY OF CARBON DIOXIDE.

The relative density of carbon dioxide compared with air as a standard is to be measured. The method employed is that used in Exp. 8. Using the same symbols as there used, and making the weighings and noting the pressures as there indicated, we have for the air,

$$(1) \qquad d_1 = \frac{mP_1}{V(P_1 - P_2)}.$$

If the measurements are then repeated for the carbon dioxide,

$$(2) \qquad d'_1 = \frac{m'P'_1}{V(P'_1 - P'_2)},$$

the symbols having the same meaning as in the case of air. From (1) and (2), if D is the relative density of the carbon dioxide, we get, by division,

$$(3) \qquad D = \frac{d_1'}{d_1} = \frac{m'P_1'\,(P_1 - P_2)}{mP_1\,(P_1' - P_2')};$$

from which we see that a determination of the volume of the flask is unnecessary.

(*a*) Read the directions given under Exp. 8. Ask an assistant for instructions in the use of the pump. Carefully dry the flask, and fill it with dry air admitted through the calcium chloride tube. Using a sensitive balance, weigh the flask full of air at atmospheric pressure, noting the pressure and temperature. In weighing, follow the method given in Exp. 1.

(b) Pump the air out until a low pressure is obtained and weigh the flask again at the reduced pressure. If the temperature is not the same, within 0°.5, the observed pressure should be corrected as in Exp. 8.

(*c*) Fill the flask with dry carbon dioxide at atmospheric pressure. This can best be done by pumping out the flask and admitting the gas from the generator several times in succession. Take care not to allow any air to pass through the acid into the generator; and keep the stop-cock closed when not using the generator. When the flask is filled with carbon dioxide at a known pressure and temperature, weigh it as before.

(*d*) Pump the carbon dioxide out until a low pressure is obtained, as in the case of the air, and weigh again.

(*e*) By the use of equation (3), calculate from your results the relative density of carbon dioxide with respect to air, under the given conditions of temperature and pressure prevailing in the room. How would you proceed to apply Boyle's and Charles' laws to the result in order to find what the ratio would be under standard conditions?

10. UNIFORMLY ACCELERATED MOTION.

The purpose of this experiment is to determine the acceleration of a freely falling body from the trace made by a vibrating tuning fork in touch with the body while falling.

Conceive of a body moving in a straight line with uniformly accelerated motion, being at a point A_0 at a certain time, A_1 one interval of time later, A_2 at the end of the second interval of time, etc. Let s_1 be the distance covered during the first interval of time, s_2 the distance covered during the second interval, etc.; and let t be the number of seconds in the given interval of time. Let v_1 be the average velocity of the body during the first interval, v_2 that during the second interval, etc. Then, by the definition of average velocity, we have

$$(1) \qquad v_1 = \frac{s_1}{t}, \quad v_2 = \frac{s_2}{t}, \quad \text{etc.}$$

Let a_1 be the average acceleration between the first and second intervals of time, a_2 that between the second and third intervals, etc. Then, by the definition of average acceleration, we have

$$(2) \qquad a_1 = \frac{v_2 - v_1}{t}, \quad a_2 = \frac{v_3 - v_2}{t}, \quad \text{etc.};$$

or substituting from (1), we get

$$(3) \qquad a_1 = \frac{s_2 - s_1}{t^2}, \quad a_2 = \frac{s_3 - s_2}{t^2}, \quad \text{etc.}$$

If the body has uniformly accelerated motion, a_1, a_2, etc. must be equal.

(If the motion is uniformly accelerated, the velocity will increase at a constant time-rate. Then v_1, the average velocity for the first interval of time, will be equal to the instantaneous velocity of the body at the middle instant of that interval; similarly v_2 will be equal to the instantaneous velocity at the middle instant of the second interval of time, etc. Between the middle instants of any two successive time-intervals the time elapsing is evidently equal to t. The average acceleration, then, between the middle instants of the first and second intervals is $(v_2 - v_1)/t$, as given above.)

(*a*) There are two forms of apparatus used in the laboratory for this experiment. In one form, the falling body con-

sists of a brass frame which falls about 120 cm. along vertical guides which offer very little friction. This frame carries with it a tuning fork, one prong of which is provided with a stylus which traces a wavy line upon the whitened glass plate clamped vertically in the support. The release of the fork by the lever at the top causes the prongs to vibrate.

In the other form, the fork is stationary and the glass plate, upon which the trace is to be made, falls about 50 cm. along the vertical guides. The vibrations of the fork, in this case, are maintained electrically. In both forms, a plumb line is used to adjust the plate and guides for the fork, so that they will be accurately vertical. The plate is first covered with a thin coat of corn-starch and alcohol, which quickly dries. It is then placed in the frame and adjustments made. At least three good traces should be obtained. A fine line is next ruled along one edge of the trace; and, starting at any convenient point, points four or five vibrations apart are marked off and their distances apart, s_1, s_2, etc., measured. Tabulate these values of s and their successive differences. Repeat for points ten vibrations apart. These measurements should be made for at least two traces.

(*b*) From the known value of the frequency of the fork find t. Calculate the acceleration for each set of observations and take the mean. Is it constant? Estimate the precision of measurement of the result. Name the principal sources of error.

11. CENTRIPETAL FORCE.

References.—Duff, pp. 24, 35; Millikan, p. 100.

The object of this experiment is to determine the force necessary to keep a body of given mass in a circle of given radius, while it moves with constant speed. Experience shows that a body in motion will continue to move with the

same speed in the same straight line, unless acted upon by some outside force. An outside force, if acting in the direction of the motion, will cause a change in speed; if acting at right angles to the direction of motion, it will cause no change in speed, but will cause a change in the direction of the motion. A body in motion always moves in a straight line, unless there is a force applied causing it to leave the straight line. If the force perpendicular to the line of the motion be momentarily supplied, the direction of the motion is changed, but the body continues to move in a straight line at an angle with its former direction. If the force be continuously supplied, the body moves in a curved path. If the body be kept in a circular path, a force of definite magnitude must be continuously applied to the body, the direction of the force being always perpendicular to the instantaneous direction of the motion. Since the instantaneous direction of motion is along the tangent, the force perpendicular to the direction of motion must be along the radius of the circle. If the force ceases to be supplied, the body ceases to leave the straight line and hence continues to move in the tangent to the circle at the position occupied by the body at the instant the force ceased to act. This is illustrated by whirling a stone at the end of a string—the string supplies the force necessary to keep the stone in a circular path. If the string breaks, the necessary force is no longer supplied, and the stone is no longer pulled out of the straight-line path. It moves away, therefore, along a tangent to its former circular path. This central force is called the *Centripetal Force,* or the *Normal Force.* It is called the normal force because it is always normal to the curved path. It is always directed toward the concave side of the curve. If the path is a circle, it is directed inward along the radius. The acceleration which it, as an unbalanced force, gives the body is also inward along the radius and is called *Normal Acceleration.*

For a circular motion the magnitude of the normal acceleration is equal to v^2/r, where v is the speed of the body and r is the radius of the circle. By the Force Equation an unbalanced force acting upon a body is proportional to the product of the mass m of the body and the acceleration produced. We have, then,

$$\text{Normal acceleration } (a_n) = \frac{v^2}{r},$$

$$\text{Centripetal force} = kma_n = km\frac{v^2}{r}.$$

If the quantities m, v, and r are expressed in C. G. S. units the factor k will be unity and the force will be given in dynes.

(a) To a rotator is attached the "centripetal force" apparatus. Two masses, m_1 and m_2, are arranged to slide along the horizontal guides. They are attached, by means of cords passing over pulleys, to a large mass M, which can slide up and down along the vertical rod. As the speed of rotation is increased, more and more force must be supplied to m_1 and m_2 in order to hold them to a circular path. Finally, when the speed passes a certain value, the force necessary to keep the masses moving in their circular paths is greater than the weight of M can supply, so the mass M is lifted.

The speed may be so regulated that M remains about halfway up the rod, or better, slowly rises and falls past this point. Its weight, Mg dynes, represents the normal or centripetal force supplied to the masses m_1 and m_2. Write the equation representing this relation. The masses M, m_1, and m_2, must be determined, and the distances of m_1 and m_2 from the axis of rotation. The speeds of m_1 and m_2 may be calculated, provided the number of rotations in a given time be counted. Make several trials, selecting each time a different set of values of the masses or of their distances from the axis. In each case maintain the speed for five minutes or more.

(*b*) For each trial, test the equality of the weight Mg and the calculated centripetal force required, and determine the percentage difference. Point out the principal sources of error in the experiment.

In the case of a body in circular motion what term is commonly applied to the *reaction* against the centripetal force? Does it act on the body, or not?

In the case of a skater describing a circle on ice, what supplies the needed centripetal force? Are the radial forces acting on the skater's body balanced? Are the vertical ones balanced?

12. THE PRINCIPLE OF MOMENTS.

References.—Millikan, p. 29; Duff, pp. 78-81.

The purpose of this experiment is to determine the condition which must be satisfied if a body, acted upon by three or more forces in the same plane, is to remain in equilibrium with reference to rotation. In order that a body at rest shall remain at rest, or a body in motion remain in motion with constant linear and angular velocity, the vector sum or resultant of all the forces acting upon it must be zero, and the algebraic sum of the moments of these forces about any axis must be zero. In the case where all the forces are in the same plane, the second of these conditions, sometimes called the Principle of Moments, requires that the sum of the moments of all the forces about any and every point selected in the plane as a center of moments shall be zero. For instance, let us suppose a case where three forces, F_1, F_2, F_3, act in the same plane upon a given body and the body remains in equilibrium with respect to rotation; if C be any point in the plane and l_1, l_2 and l_3 be the lever-arms or perpendicular distances from C to the lines of direction of the three forces, and if the moments with respect to C of two of the forces be anti-clockwise or positive and

the moment of the third force with respect to C be clockwise or negative, then the Principle of Moments requires that

$$+ F_1 l_1 + F_2 l_2 - F_3 l_3 = 0.$$

To prove the principle it is only necessary to show that the sum is zero for one selected point, provided that this point is so chosen as not to lie in the line of any of the forces. If a point in the line of any force were chosen, the moment of that force with reference to that point as a center of moments would be zero no matter what the value of the force; hence, the result for such a choice, would not be a test of the principle.

The apparatus used to test the principle consists of a circular table with a movable disk resting on bicycle balls. The disk may be pivoted in the center if desired. To pegs, placed at will in the disk, cords are attached which pass over pulleys clamped at different points around the circular table. From the ends of the cords are suspended known masses whose weight produces the forces required.

(a) Place the disk on four bicycle balls, widely separated, and level up the table so that the disk will not tend to move in any one direction in preference to another. Pivot the disk in the center and place a sheet of manila paper upon it. Attach cords to it at three different points chosen at random; and, placing the pulleys at any convenient points, add masses until the three forces are of convenient values. See that the disk is free to move on the bicycle balls and that the cords all lie in a plane close to and parallel to the top of the disk; then mark points or lines on the paper to indicate the directions of the forces. Note the magnitude of the forces, counting in the weight of the pan in each force.

(*b*) Remove the paper, trace the lines of direction of the forces, and make the measurements necessary to determine their moments about the pivot as an axis. Find the sum of

the moments, taking those as positive which tend to produce a counter-clockwise rotation about the given axis, and those as negative which tend to produce a clockwise rotation.

(*c*) Choose at random any point on the paper used in (*a*) and (*b*), and find the sum of the moments about this point as a center of moments. Why is not the sum zero?

(*d*) Remove the pivot, and repeat (*a*) and (*b*) once, selecting in turn as centers of moments three points as widely separated as possible. Keep the disk free of the rim about it, so that the only forces acting on the disk in the horizontal plane are those due to the cords. Find the sum of the moments for each of these centers as before. Also find the vector sum or resultant of the forces by the method of the closed polygon.

(*e*) Repeat (*d*), using four forces instead of three.

(*f*) From the data of (*a*) and (*b*) determine the vector sum of the three forces used in that case. If this sum is not zero, it means that the pivot itself exerted a force on the disk in the same plane with the three forces. What do you conclude is the magnitude and direction of this force? Draw its line of direction on the paper, and then repeat (*c*), including now in your sum the moment of the force due to the pivot. Is the sum now approximately zero?

(*g*) In the various cases of equilibrium considered above, what do you find the vector sum of the forces to be? 0 What have you found to hold true for the moments of these forces? 0 Calculate the percentage error for one case.

13. THE SIMPLE PENDULUM.

References.—Duff, p. 87; Millikan, p. 95.

The purpose of this experiment is to determine the acceleration due to gravity from a knowledge of the period and length

of a simple pendulum. For vibrations of small amplitude the period of such a pendulum is given by the equation,

$$T = 2\pi\sqrt{\frac{l}{g}},$$

where T is the time of one complete vibration, l is the length of the pendulum, and g is the acceleration due to gravity. If T and l are known for any place, g can be determined for that place.

Method of Coincidences.

In the present experiment, T is to be measured by comparing, by the "method of coincidences," the period of the simple pendulum with that of a clock pendulum of known period. An electric circuit is completed through an electric bell, the clock pendulum, the simple pendulum, and the mercury contacts at the bottom of each pendulum. Assume that the period of the clock pendulum is two seconds, that is, that the time of a single swing or half-vibration is one second. If the period of the simple pendulum were the same and the two pendulums be started together, they would vibrate in coincidence and the bell would ring with every passage. If, however, the time of a single swing of the simple pendulum were less than one second, say by $1/n$ th of a second, it would gain on the clock pendulum and thus fall out of coincidence with it, so that the bell would cease to ring until n seconds later, when the two pendulums would be in coincidence again. Let us suppose that the time between these successive coincidences is 100 seconds, then we know that in this time the clock pendulum has made one hundred half-vibrations and the simple pendulum one more, or 101 half-vibrations. In other words, the simple pendulum has made 101 half-vibrations in 100 seconds, hence the value of its half-period is 100/101 seconds. If, on the other hand, the simple pendulum had been observed to lag behind the clock pendulum, and the time between suc-

cessive coincidences remained the same, we would know that its half-period is 100/99 seconds.

(*a*) The simple pendulum used consists of a brass sphere suspended from a knife-edge by a wire so that the length is adjustable. The mercury contact below should be so adjusted that the platinum point on the ball passes freely through it Adjust the pendulum so that its length is either greater or less, by 2 or 3 cm., than that of a pendulum beating seconds. Two different lengths (in successive determinations) should be used such that one is greater and the other less than that of a pendulum beating seconds. In getting the length it is well to measure with a meter rod and square to the top of the ball, and then to determine the diameter of the ball with the calipers. The length of the pendulum is the distance from the knife-edge to the center of the ball. After adjusting the mercury contact, start the ball swinging in an arc of about 10 cm., taking care not to give it a twisting vibratory motion. During the vibrations watch the hands of the laboratory clock and record the hour, minute and second of each successive coincidence between the simple pendulum and the clock pendulum, up to ten or more. If the bell rings for more than one swing during each coincidence, take the mean of the times of the first and last rings as the time of the coincidence.

(*b*) To obtain from the data a more reliable average value of the time between successive coincidences, proceed as follows: Find the difference in time between the first and sixth coincidences, the second and seventh, and so on, and take the mean. From this the average time between successive coincidences may be found and the period calculated. Be sure to note whether the pendulum was gaining or losing on the clock. Calculate the value of g for the two cases and take the mean.

(*c*) What effect would be produced upon the vibration of a pendulum by carrying it, (1) to a mountain top, (2) from the equator to the pole of the earth? In what way does the

pendulum used in this experiment fall short of the requirements for a simple pendulum? What is the object of taking a small amplitude of vibration?

14. THE FORCE EQUATION.

References.—Duff, p. 31; Millikan, p. 15.

If F is the resultant force acting on a body, m its mass, and a the acceleration produced, we have, as a result of definition and experiment,

$$(1) \quad F = k\ m\ a.$$

This equation is called the Force Equation, or Equation of Motion, and the purpose of the experiment is to verify it. The factor k in the equation is a numerical constant whose value depends upon the system of units used. This equation states (1) that, if two forces act on bodies of the same mass, the accelerations produced will be directly proportional to the forces; and (2) that, if two forces produce the same acceleration in two bodies of different mass, the masses will be directly proportional to the forces. Let M, M be two equal masses suspended from a cord passing over a pulley whose friction and rotational inertia we will assume to be negligible. The total mass suspended is $2M$; the resultant force acting upon it is zero. Let a mass m be removed from one side. The resultant force F_1 now is $k\ m_1\ g$, and it will cause the mass $(2M - m_1)$ to move in its direction with an acceleration a_1; hence, by equation (1),

$$(2) \quad F_1 = k\ (2M - m_1)\ a_1.$$

If a different force be applied by changing to m_2 the value of the mass removed, the resultant force (F_2) will be $k\ m_2\ g$,

and it will produce an acceleration a_2; hence, by equation (1),

$$(3) \qquad F_2 = k\,(2M - m_2)\,a_2.$$

Hence
$$\frac{k\,m_1\,g}{k\,m_2\,g} = \frac{F_1}{F_2} = \frac{k\,(2M - m_1)\,a_1}{k\,(2M - m_2)\,a_2}, \quad \text{or}$$

$$(4) \qquad \frac{m_1}{m_2} = \frac{(\,2M - m_1\,)\,a_1}{(\,2M - m_2\,)\,a_2}\,.$$

An experimental verification of equation (4) will constitute a verification of equation (1), though it will not, of course, determine the value of the constant k.

The apparatus used in Exp. 10 is employed, with the addition of a pulley-attachment at the top over which a cord passes, from one end of which the fork or the glass plate (dependent upon which form of apparatus is used) is suspended and from the other end a number of masses just sufficient to balance the same and the friction of the pulley. Note the precautions given in Exp. 10. Special care should be taken to insure as little friction as possible.

(*a*) Adjust the apparatus so that a good trace may be obtained and so that a slight tap will cause the fork or the glass plate to descend without acceleration. The forces, including friction, are then just balanced. Cover the plate with a thin coat of corn-starch and alcohol. Take care to have the stylus exert the same pressure against the plate throughout the experiment.

(*b*) Remove a mass m_1 from the balancing masses. Note the total mass $(2M - m_1)$ of the moving system. Obtain two good traces.

(*c*) Repeat with a different mass m_2 removed, the total mass of the system now being $(2M - m_2)$.

(*d*) Repeat again with a third mass removed.

(*e*) Measure the traces as explained in Exp. 10, using five vibrations of the fork as the interval of time. Calculate

$2\,m\,a_2$

$2\,T$

the accelerations a_1, a_2, a_3, corresponding to (b), (c), (d) above. Then make two tests of equation (4) by substituting in the same. Calculate the percentage difference between the two sides of the equation in each case.

If the masses removed in (b), (c), (d) had simply been transferred from one side of the pulley to the other, what changes would be required in substituting in equation (4)?

15. SURFACE TENSION BY JOLLY'S BALANCE.

References.—Duff, p. 146; Millikan, p. 181.

The purpose of this experiment is to obtain a direct measure of the surface tension of a liquid by balancing it against the tension in a stretched spring. A wire rectangle is hung from the spring of a Jolly's balance and allowed to dip in a soap solution which forms a film across the rectangle. When equilibrium is established the force due to surface tension in the two surfaces of the film must just balance the tension in the spring. By knowing the force which will stretch the spring the same amount, we have a measure of the total force due to surface tension. If T is the value of the surface tension per centimeter width of the film, l the width of the rectangle along the surface of the liquid, and F the force exerted by the spring, then it follows, because the forces are in equilibrium, that

$$F = 2\,l\,T$$

Knowing F and l, the value of T can thus be found.

The Jolly's balance used is one of the two forms used in Exp. 2. Ask for directions, if its operation is not already understood. Wire rectangles of different sizes and a wide beaker are provided. The greatest care must be taken that the beaker and rectangles are clean. They should be washed in caustic potash and rinsed thoroughly in hot water before being used and before changing to another liquid. Do not touch

with the fingers the inside of the beaker, the liquid, or the part of the rectangle on which the film is formed.

(*a*) Suspend a rectangle, 2 cm. wide, from the spring, and let it be partially immersed in a beaker of soap-solution. Read the extension of the spring when there is no film in the rectangle, and again with a film across it. The rectangle should be immersed to the same depth in the two cases, so as to eliminate the effect of the buoyancy of the liquid. Take three sets of readings. Note whether the pull of the film depends upon the area of it formed in the rectangle.

Repeat these measurements, using rectangles 4 cm. and 6 cm. wide.

(*b*) Calibrate the balance by observing the extension produced by known standard masses.

(*c*) Use the rectangle, 4 cm. wide, cleaning it and the beaker thoroughly, and repeat (*a*) with water fresh from the tap. As a film of no appreciable height will form with pure water, take the reading of the balance without the film when the under side of the upper wire of the rectangle is just above the surface of the water and not in contact with it; and again, after immersing the upper wire of the rectangle so as to wet it, take a reading when it breaks away from the surface. Take three sets of readings.

(*d*) Repeat (*c*), using water at 50°C. or higher.

(*e*) Repeat (*c*), using alcohol.

(*f*) From the data taken in (*a*), state how the total tension in the film varies with its width. Calculate the surface tension, T, in dynes per cm., for the liquids used in (*a*), (*c*), (*d*), and (*e*), comparing the values obtained and pointing out how the surface tension is affected by the temperature.

16. CAPILLARITY. RISE OF LIQUIDS IN TUBES.

Reference.—Duff, p. 149.

In the present experiment the values of the surface tension of water and of alcohol are to be measured by observing the

rise of these liquids in capillary tubes. When the inner surface of a tube is wet by a liquid, the surface tension of the latter may be considered as acting upward at all points around the circumference of the tube. The total vertical component of this force is $2\pi\ r\ T \cos \alpha$, where r is the radius of the tube, T the surface tension in dynes per cm., and α is the angle of contact between the liquid and the tube. If the tube is of small bore, the liquid will rise inside the tube, equilibrium being established when the weight of the liquid within the tube above the level of the liquid outside equals the vertical force upward due to surface tension. If d is the density of the liquid, h its height in the capillary tube above the surface level, and g the acceleration due to gravity, it follows that

$$\pi\ r^2\ h\ d\ g = 2\ \pi\ r\ T \cos \alpha.$$

From this equation the value of T, the surface tension in dynes per cm., can be found.

(*a*) Capillary tubes of different sizes are provided. These may be thermometer-tubes or larger glass tubing drawn out to a fine bore. In either case every precaution must be taken to have the tubes perfectly clean and free from all traces of grease. They should be cleaned with caustic potash solution, rinsed with tap water and then with the liquid to be experimented with (in this case, water). With a rubber band fasten the tubes side by side to a glass scale, and place the scale and tubes vertically in a small dish of distilled water. Lower the tubes first to the bottom of the dish so as to wet the inside for some distance above the point to which the water will rise. Then clamp them with the ends below the surface, and note on the scale the point to which the water rises in each tube. To obtain the reading for the water-surface in the dish a wire hook is provided, which should be brought up so that the point is just even with the surface. Then read the height of this point on the glass scale.

(*b*) Measure the inside diameter of the tube with a micrometer microscope. If drawn-out tubing is used, scratch the tube with a file at the point to which the water rises, break it and measure the diameter of the end. If the tube is uniform in bore, its diameter can be found either with the micrometer microscope, or by means of a thread of mercury drawn into the tube. In case the latter method is used, the length and mass of the thread and the density of mercury are all the data needed for calculating the diameter. Calculate the surface tension of water, and compare this value with that found in Exp. 15. For pure water and ordinary glass the angle of contact is approximately zero.

(*c*) In the same way find the surface tension of alcohol. For the angle of contact in this case see the Tables.

Would the water or alcohol rise as high in the tubes if the experiment were performed in a vacuum? Explain.

If a thread of water were placed in a horizontal, conical-shaped tube, in which direction along the tube would it move? Explain. If mercury instead of water were used, what would happen, and why?

17. RISE OF LIQUIDS BETWEEN PLATES.

Reference.—Hastings and Beach, p. 146.

In the present experiment the surface tension of water and of alcohol is to be measured by means of the rise of the liquid in a wedge-shaped space between two plates of glass. The two plates of glass, which are in touch with each other on one side, are separated on the other side by a thin piece of brass placed between the opposite edges of the plates. The plates are clamped together and placed upright in a shallow dish of liquid. If the liquid wets the plates, it will rise in the wedge-shaped space, forming a smooth curve which extends from the surface of the liquid in the dish, on the side where the

plates are farthest apart, to a point high above this level, on the other side where the plates are in touch with each other. The general effect is similar to that obtained by a row of small tubes of gradually decreasing bore. We may consider that at some point along the curve a thin vertical slice or rectangular prism of the liquid is taken. Let d, the distance between the plates at the point chosen, be the width of the prism; x (very small), the thickness of the prism in a direction parallel to the plates and to the surface of the liquid in the dish; and h the height of the prism above the surface of the liquid in the dish. The surface tension which acts upon this prism evidently has a vertical component upward equal to $2\ T\ x$, where T is the value of the surface tension in dynes per cm. This force must equal the weight of the prism of liquid which is $h\ x\ d\ D\ g$, where D is the density of the liquid and g the acceleration due to gravity. From this relation T can be found.

(a) Clean the plates very carefully with caustic potash solution, and rinse with water. Clamp them together as indicated above, and upon one side of one of the plates place a thin sheet of white paper. Stand the plates upright in a shallow vessel of distilled water, and looking through the paper and the plates toward the light, trace on the paper the surface of the water between the plates, the surface of the water in the dish, the outline of the piece of metal, and the edge where the plates touch each other. (If the water-curve between the plates is not a smooth one, it will be necessary to raise and lower the plates in the dish until the surfaces of the glass between them is thoroughly wet.) Removing the sheet of paper, draw a line on the paper to show the position of the inner edge of the piece of brass. This line, as well as the line showing the position of the edge where the plates were in touch with each other, should be perpendicular to the line representing the surface of the water in the dish. Select any point P on the curve representing the surface of the water between

the plates. From this point draw a line perpendicular to the line representing the surface of the water in the dish, and call its length h. Let x be an infinitesimal distance through P at right angles to this last line. To determine the width d between the plates at P, proceed as follows: Draw a line through P parallel to the line representing the surface of the water in the dish and let the length along this line from P to the line showing where the plates were in touch with each other be l. Let the whole distance from the inner edge of the piece of brass to this same line be L. Measure the thickness d_1 of the piece of brass with a micrometer caliper. Then, at the point P,

$$d = \frac{l}{L} \cdot d_1.$$

Derive this equation. From the values of d and h thus found, calculate the surface tension of water in dynes per cm. Repeat the measurements and calculation for one or two other points on the curve.

(b) Repeat (a), using alcohol instead of water, and find the surface tension of alcohol.

18. VISCOSITY. FLOW OF LIQUIDS IN TUBES.

Reference.—Duff, p. 137.

The dependence, of the rate of flow in tubes, on the diameter and length of the tube, and on the temperature of the liquid and the kind of liquid used, is to be observed. When a liquid flows through a tube, if the liquid wets the walls of the tube, the layer of liquid in immediate contact with the wall generally remains at rest. The speed with which the liquid moves increases from the surface of the tube to the axis of the tube. Hence, if we imagine the liquid to consist of a number of hollow cylinders coaxial with the tube, the fluid within

each of these cylindrical shells will be moving more slowly than in the shell immediately inside, and more rapidly than in the shell immediately outside. This relative motion of adjacent layers of the liquid is determined by the internal friction or viscosity of the liquid. Viscosity varies greatly with the kind of liquid used, this dependence upon the character of the liquid being indicated by the *coefficient of viscosity.* If a liquid is very viscous, like syrup, its coefficient of viscosity is high; if like alcohol, its coefficient of viscosity is low. For a given liquid at a given temperature, the coefficient of viscosity is a constant.

In the case of a liquid flowing through a long, narrow tube, the volume V, issuing per second from the end, depends upon the difference in pressure p, between the two ends of the tube, the radius r of the tube, its length l, and the coefficient of viscosity c of the liquid. These quantities are connected by the relation

$$V = \frac{\pi p r^4}{8lc}.$$

To compare the coefficients of viscosity of two different liquids, it is evident, if the above relation be accepted, that, for the same tube and equal times of flowing, the coefficients will be in inverse proportion to the volumes, or $c_1 : c_2 = V_2 : V_1$.

Three small-bore tubes are provided, two being of the same length but of different bores, and the third being longer but of the same bore as one of the two shorter ones. The reservoir used consists of a large bottle through whose cork are fitted two glass tubes, long enough to reach about two-thirds of the way to the bottom. The outside end of one of these tubes is connected by rubber tubing with the tube through which the flow is to be measured; the other tube is left open to the air. Both tubes must extend some distance below the level of the liquid in the bottle, and the cork must be air-tight. By

means of this arrangement a constant head of pressure may be obtained. The tube, which carries the liquid from the reservoir to the small-bore tube, is quite large, so that the frictional resistance which it offers to the flow will be negligible as compared to that offered by the small tube. This makes it reasonable to assume (as is done in the experiment) that the head of pressure is all employed against the frictional resistance offered by the small tube.

(*a*) Clean the tubes thoroughly with chromic acid, and rinse by drawing clean water through them with a jet-pump. Attach one of the tubes to the siphon-tube from the reservoir, letting the lower end dip into water in a beaker. Weigh the beaker and contained water on the trip-scales. Before replacing the beaker in position, nearly fill the reservoir with water at the room temperature, start the siphon, and let the water run into a waste vessel until the air begins to bubble from the lower end of the open tube up through the water in the reservoir. Then replace the beaker, record the height of the water-level in it, and allow the water to flow for two minutes. Weigh the beaker again to determine the volume which has run through. The head of pressure will be given by the difference in height of the lower end of the open tube in the reservoir and the mean of the initial and final levels in the beaker. Point out clearly why the head is measured from the end of the open tube and not from the water-level in the reservoir. Make two independent trials.

(*b*) Repeat with each of the other tubes. Measure the diameters of the tubes with the micrometer microscope, or by weighing mercury which occupies a known length of the tube. What do your results show concerning the dependence of the rate of flow on the radius and length of the tube? Employing the C. G. S. system of units, calculate the coefficient of viscosity of the water for the three cases, and take the average value.

(*c*) With one of the tubes, use water at 50°-60°C. in the reservoir, and compare with previous results to determine the effect of temperature on viscosity.

(*d*) Repeat (*c*) with a ten-per-cent solution of sugar, and, if there is time, with a ten-per-cent salt-solution. Discuss the results, comparing them with those of (*a*) and (*b*), noting the effect upon viscosity of different sorts of dissolved substances.

19. EFFLUX OF GASES. RELATIVE DENSITIES.

Reference.—Duff, p. 166.

The object of this experiment is to find the relative densities of certain gases from the observation of the relative times of efflux of equal volumes of these gases through a small aperture. The ratio of the densities of two gases, under the same conditions as to pressure, is equal, very approximately, to the inverse ratio of the squares of the speeds with which the gases escape through a fine opening in a diaphragm. Since the time of escape of a given volume will be inversely as the speed of efflux, it follows that the ratio of the densities of two gases is equal to the direct ratio of the squares of the time of efflux of equal volumes under the same conditions. This relation was experimentally discovered by Bunsen. For a proof of it, from the energy relations, see the reference given above.

(*a*) The gas-holder consists of a glass cylinder, at the top of which is a three-way stop-cock and a diaphragm with a fine opening. The cylinder is placed in a reservoir of mercury. The three-way cock allows communication to be made with the outside for filling or with the diaphragm. Within the cylinder is a float which indicates when the desired volume of gas has escaped.

First fill the cylinder with dry air. To do this, turn the

stop-cock so as to put the cylinder in communication with the air, and lower the cylinder as far as it will go. This drives out most of the contained gas. Connect the cylinder with a calcium-chloride drying-tube, and raise the cylinder. This operation will fill the cylinder, and by repeatedly emptying and filling the cylinder it will become practically freed of the moist air or other gas previously contained in it. Close the stop-cock, and lowering the cylinder, clamp it in position Turning the stop-cock so that the gas in the cylinder is in communication with the diaphragm, note the time when the upper point of the float is on a level with the surface of the mercury or with a mark on the cylinder. Again note the time when the second mark on the float is on the same level. Repeat, making two or three determinations of the time of efflux for the given volume of air, and take the mean.

(*b*) Repeat (*a*), filling the cylinder with illuminating gas, following the directions there given for filling the cylinder, the cylinder being connected directly to the source of the gas used. Note the time of efflux between the same two positions for the float as used in (*a*). This insures the same conditions as to pressure in the two cases.

(*c*) Repeat (*b*), using dry carbon dioxide.

(*d*) Calculate the relative densities, referred to air, of the gases used in (*b*) and (*c*). Taking the density of dry air under standard conditions to be 0.001293 gms. per cc., find the density, under standard conditions, of the gases used. What "Laws" have been used, or assumptions made, in answering the requirement of the preceding sentence?

20. ABSOLUTE CALIBRATION OF A THERMOMETER.

References.—Watson's Practical Physics, p. 162; Edser, p. 23.

The object of this experiment is to plot a curve from which the true temperature may be obtained corresponding to each

scale-reading of a given mercurial thermometer. Such a curve is called the *calibration curve* of the thermometer. The process of obtaining it is *absolute* since it does not involve comparison with a standard thermometer.

(*a*) *Correction near the Lower Fixed Point.*—Put the thermometer through the cork in a test-tube, having filled the latter about half full of distilled water. Place the tube in a freezing mixture of shaved ice and salt, and stir the water around the thermometer until it begins to freeze. Read the thermometer. By warming the tube in the hand and repeating the freezing process, obtain several readings. Let us suppose that the mean of these readings is + 0.2°C. Since the true temperature of freezing water is 0°C., the correction corresponding to the given scale-reading of the thermometer is — 0.2°, for this when added to the reading gives the true temperature.

(*b*) *Correction Near the Upper Fixed Point.*—Place the thermometer through the cork in the tube at the top of the boiler, with the bulb well above the surface of the water. Boil the water so that the steam issues freely, but not with any perceptible pressure, from the upper vent. Read the thermometer when it becomes steady. Allow the boiler to cool slightly, and repeat, making three readings in all. If the instrument be provided with a water-manometer, take the manometer-reading simultaneously with the temperature-reading. Read the barometer and determine the pressure of the steam, and find from the Tables the true boiling-point temperature for this pressure. Let us suppose that the mean of the readings of the thermometer is 99.1°C., while the true temperature is 99.8°C. Then the correction corresponding to this scale-reading of the thermometer is +0.7°, for this when added to the reading gives the true temperature.

(c) Let the thermometer cool slowly to about the temperature of the room, and repeat (*a*). If the freezing point ob-

served now is different from that observed in (*a*), use the mean of the two values in the calibration that follows. Assuming the temperature of freezing water to be 0°C., write the corrections of the thermometer for the scale-readings observed in (*a*) and (*b*). Record these two corrections by points on coördinate paper, having as abscissae the scale-readings of the given thermometer from 0° to 110°, and as ordinates the corresponding corrections in tenths of a degree but on a larger scale. Corrections should be plus (+) if they are to be added to the observed to give the true temperatures, minus (—) if they are to be subtracted. Connect these two points by a straight line. The ordinate of this straight line at any point gives the correction of the thermometer at that scale-reading on the assumption that the bore of the thermometer is uniform throughout the whole range. In general this assumption is not justified, and there must be added to this correction at each point another correction due to the inequalities of the diameter of the bore. In order to determine this latter correction, it will be necessary to calibrate the tube.

(*d*) *Calibration of the Tube.*—Break off a portion of the thread of mercury about ten degrees in length. Ask for assistance, if necessary. Place the lower end of the thread, approximately ten degrees long, at the zero-point of the scale and read the position of the upper end to tenths of a degree. Then place the lower end at 10° and read the position of the upper end. Repeat with the lower end at the successive points 20°, 30°, 40°, etc., up to 90°; then come down again with upper end at 100°, 90°, 80°, etc., reading the position of the lower end each time.

(*e*) Record the observations and calculations in tabular form in six columns as follows:

(1) The reading of the lower end of the thread.
(2) The corresponding reading of the upper end.
(3) The length of the thread in each position.
(4) The mean length l for each interval.

By the mean length for each interval is meant the mean of the reading over a certain interval going up (say from 30 to 40) and over the same interval (40 to 30) coming down. Find the mean value of all these mean lengths throughout the whole range and record this as the mean length L of the thread for the whole range. 11.06

11.06 - l

(5) The correction, $L—l$, for the length of each interval, that is, the difference between the mean length for all intervals and the observed length for each interval.

(6) The correction for the upper end of each interval. This is the correction for the lower end of the interval plus the (correction for the length of the interval) (l - L) since a correction at any point evidently affects all points above this. The correction thus found for any point represents the magnitude of the inequalities of the bore up to that point. It must be added to the observed reading for that point to give the correct reading. The corrections should be recorded with proper signs (See Watson's Practical Physics, p. 168.)

(*f*) To construct a final table of corrections it is necessary to add, algebraically, the corrections found in (*c*) and in (*e*, 6). This can best be done by plotting. On the plot made in (*c*), plot points whose abscissae are 10°, 20°, 30°, etc., and whose corresponding ordinates are found by measuring from the slanting line, already drawn, distances equal to the corresponding corrections found in (*e*, 6)—measuring up or down from this line according as the corrections are plus or minus. The smooth curve, which should now be drawn through these plotted points, is the calibration curve of the thermometer.

What are the temperatures corresponding to readings of 0°, 25°, 50°, 75° and 100° on the given thermometer?

21. RELATIVE CALIBRATION OF A THERMOMETER.

Most varieties of glass expand at different rates at different temperatures, hence, even with a thermometer whose bore has been carefully calibrated by some such method as given in Exp. 20, the reading can be relied upon only within certain limits. After having obtained a thermometer whose calibration curve is accurately known, so that it may be taken as a "standard," the most convenient method of calibrating other thermometers is by direct comparison with the standard, hence the name "relative calibration." If the calibration curve of the standard thermometer can be relied upon, all irregularities of any other thermometer can be corrected.

The thermometer to be calibrated in this experiment is a 50° thermometer reading to tenths of a degree. Tie the thermometer to the standard thermometer with soft cotton twine, winding it between the stems so as to separate them slightly. Put the bulbs nearly opposite each other; and see that corresponding divisions are as nearly opposite as is consistent with this condition. Suspend the two securely, with the bulbs in the middle of a kettle of water, and steady the stems by catching them loosely, without pressure, in a clamp. The thermometers are to be read by a short-focus telescope, which slides easily on the vertical rod of its stand. This should be set with its object-glass at a distance of about 50 cm. from the thermometers, which should be perpendicular to its axis. When taking a reading, always set the telescope so that the top of the mercury column appears in the middle of the field of view (not near its upper or lower edge) in order to avoid parallax.

(*a*) Take a careful series of readings, to hundredths of a degree, at intervals of 2° or 3° from about 5° to 45°. Keep the water well stirred, and keep the temperature fairly con-

stant for a few minutes before each reading. A good plan is to take a preliminary reading of each thermometer in order to see about where the reading is going to come. The two exact readings can then be made so quickly as to be practically simultaneous. Read again in a few seconds, taking the thermometers in reverse order. Repeat, if necessary, until the differences obtained for two such readings agree fairly well.

(b) Let the observers change places, and take a similar descending series, cooling the water by dipping out hot and adding cold water.

(*c*) Ask to see the calibration curve of the standard used, and from it construct a table of corrections for the thermometer you are calibrating. Plot a calibration curve, recording the number of the thermometer. In your future work with a thermometer of this type, use the one you have calibrated.

22. VARIATION OF BOILING POINT WITH PRESSURE.

Reference.—Duff, p. 229.

There are two methods employed in studying the variation in the boiling point of a liquid with the pressure upon its free surface. By the dynamic method the pressure above the boiling liquid is varied by means of an air-pump and the corresponding temperature observed. By the static method the temperature of the liquid, suitably enclosed, is varied by means of baths and the corresponding pressure observed. *The object of the present experiment is to study the variation in the boiling point of water, employing the dynamic method.*

The apparatus consists of an air-tight boiler to hold the liquid, a steam-condenser, around which cold water circulates, an air-tight chamber large enough to equalize sudden changes in the pressure, an air-pump for reducing the pressure and a

manometer for measuring the same. These are connected up in the order named and made air-tight so far as the air outside is concerned.

(*a*) The circulation of water should first be started through the steam-condenser, which is a glass or metal tube used to jacket the tube leading from the boiling-flask, thus condensing the steam as it comes from the flask. The thermometer should be passed through the stopper of the flask and so regulated that its bulb will be in the rising steam, and not in the water. The connection with the large glass bottle serves to equalize sudden changes in pressure due to irregularities in the boiling. In heating the water do not play the flame on the flask directly under the glass beads, but rather to one side and below the water-line.

First boil the water at atmospheric pressure, reading the manometer and noting the temperature. Then take a series of readings at intervals of about 5 cm. pressure, until the "bumping" becomes so violent as to render further readings impracticable. Before each reading, after pumping to the pressure desired, close the stop-cock over the jet-pump, wait a short time for the pressure to reach equilibrium, and then make the reading of boiler temperature and corresponding pressure. Put the pump again in connection, obtain a new pressure, and repeat the readings. Before turning off the water at the jet, be sure each time to let air into the apparatus by opening the pinch-cock nearest the pump, otherwise water will flow back into the tubing.

(b) Take a series of readings with increasing pressures up to atmospheric pressure, choosing values different from the previous ones.

(c) Plot the observations on coördinate paper, using pressures as ordinates and temperatures as abscissae. From the curve find the boiling point of water at a pressure of 1/2 atmosphere. Discuss the phenomena of this experiment in

connection with the difficulties experienced in cooking food at high altitudes. Could determinations of the boiling point of water be used to measure altitude, and how?

23. COEFFICIENT OF EXPANSION OF A LIQUID BY ARCHIMEDES' PRINCIPLE.

The coefficient of expansion of a heavy oil is to be obtained by observing the change in the buoyant force acting on a metal cylinder when immersed in the oil at different temperatures. A brass cylinder is suspended from one arm of the balance and carefully weighed, first in air, then in water at a known temperature. The oil is then placed in a calorimeter consisting of one beaker inside another, and the cylinder is weighed when immersed in the oil, the temperature of the oil being noted, which should be the same as that of the water, or nearly so. Since the oil thickens if cooled, it is convenient to make the first weighings at the room temperature.

After weighing in the cool oil, the inner beaker is removed and placed in a water-bath heated to 60° or 70°C. Replacing the beaker with the heated oil in the calorimeter beaker, the cylinder is again weighed in the oil, the temperature of the oil during the weighing being carefully noted.

Let M = the mass balancing the cylinder when in air,
m_1 = the mass balancing the cylinder when in water,
m_2 = the mass balancing the cylinder when in cool oil,
m_3 = the mass balancing the cylinder when in hot oil.
t_1 = the temperature of the cool oil and the water, and
t_2 = the temperature of the hot oil.

Of the quantities not directly measured, but which must be known in order to find the coefficient of expansion of the oil, (1) let V_1 represent the volume of the brass cylinder at the lower temperature t, and d the density of the water at the

same temperature (see the Tables). By Archimedes' principle and the definition of density

$$V_1 = \frac{M - m_1}{d}.$$

(2) Let V_2 represent the volume of the cylinder at the higher temperature t_2 and α the coefficient of cubical expansion of brass (see the Tables). By definition of α,

$$V_2 = V_1 \; [1 + \alpha \; (t_2 - t_1)]$$

(3) Let d_1 and d_2 represent the densities of the oil at the lower and higher temperatures respectively. By Archimedes' principle and the definition of density

$$d_1 = \frac{M - m_2}{V_1} \quad \text{and} \quad d_2 = \frac{M - m_3}{V_2}.$$

(4) Finally, let β represent the coefficient of cubical expansion of the oil. If any mass m of oil has a volume V' at a temperature t_1 and a volume V'' at a higher temperature t_2, then $V'' = V' \; [1 + \beta \; (t_2 - t_1)]$. Dividing both members by m, and substituting for m/V' and m/V'' their equivalents d_1 and d_2, the equation becomes $d_1 = d_2 \; [1 + \beta \; (t_2 - t_1)]$. This is true generally and therefore in the present experiment. By this last relation

$$\beta = \frac{d_1 - d_2}{d_2 \; (t_2 - t_1)}.$$

Carry out the experiment as outlined and calculate β.

Point out the sources of error. If the brass cylinder should have had an internal cavity, show what its effect upon the value of β would be.

24. COMPARISON OF ALCOHOL AND WATER THERMOMETERS.

In this experiment the relative expansions of water and alcohol are to be studied, and the behavior of these liquids when used in thermometers to be observed.

(*a*) Two thermometer-bulbs are to be filled, one with water the other with ethyl alcohol, by the aid of the reservoir-tube. The reservoir is fitted on the end of the thermometer-stem, filled with water (or alcohol), and warmed. The water should first be boiled to drive out the oxygen held in solution, before filling the reservoir with it. The liquid is then introduced into the thermometer-bulb by alternately heating the bulb to drive out the air and allowing it to cool to admit the liquid. When all but a tiny bubble of air has been removed, place the bulb in ice-water and force the liquid to dissolve the air. If this does not succeed, ask for assistance. Take care not to ignite the alcohol. The liquid in each thermometer should stand 1 or 2 cm. above the lower end of the stem when the bulb is in melting ice.

(*b*) Glue or otherwise fasten a strip of stiff paper along the back of each stem, and use it as a scale. Then place the thermometers in clamps with their bulbs in shaved ice or in a mixture of water and ice. When the reading becomes steady, indicate the position of the meniscus of each by a sharp line on the card. Mark the line zero. This is the first fixed point of the thermometer.

(*c*) To determine the second fixed point, place the bulbs in a beaker of wood alcohol which is itself placed on a support in a bath of water. Heat the water-bath slowly until the wood alcohol begins to boil. Be very careful not to bring the alcohol itself to the flame, and avoid inhaling the fumes of wood alcohol. When steady, again indicate the position of

the meniscus on each stem by a sharp line. Mark this point 66, which is the boiling point of wood alcohol on the centigrade scale.

(*d*) Lay the stems of the thermometers on a flat surface, measure the distance on each between the two fixed points, and divide this distance into 66 equal parts, calling each part a degree. Put the marks for each degree on the scale and number every tenth one.

(*e*) Place the two arbitrarily calibrated thermometers in a water-bath at 0°, as recorded by each thermometer. Gradually raise the temperature of the water-bath and note the readings of the two thermometers, at first at short intervals, then at longer intervals, until the upper fixed point is reached. Each time that the temperature is raised, it should be kept at its new value for three or four minutes, so as to give the bulbs time to assume the temperature of the bath.

(*f*) Plot the series of points on coördinate paper, having as abscissae the temperatures by the alcohol-thermometer, and as ordinates the corresponding temperatures by the water-thermometer. Draw a smooth curve through the points. This curve gives the relation between the temperatures as recorded by the two thermometers. What inferences can you draw from the curve? If alcohol be taken as the standard substance, what can you say of the uniformity of the expansion of the water? If the water be assumed as the standard, what of the expansion of the alcohol? Which would be the better substance to use in a practical thermometer, and why?

25. COEFFICIENT OF EXPANSION OF A LIQUID BY REGNAULT'S METHOD.

References.—Duff, p. 199; Edser, pp. 71, 76.

This method was originally devised by Dulong and Petit, but was improved and made practical by Regnault. It is an

absolute method in that the effect of the expansion of the containing vessel is eliminated. It is applicable to any liquid. *The purpose of the present experiment is to determine the coefficient of expansion of turpentine or olive oil by means of this method.*

Two glass tubes, *A A′* and *B B′*, are surrounded by metal cylinders, *L* and *M* respectively, in which baths at different temperatures may be placed. These glass tubes are connected near the top by a horizontal tube *A C B* from which there extends an upright open tube *C*, and are connected at the bottom between *A′* and *B′* by an inverted U-tube. The liquid is poured into the glass tubes until it stands at some point in *C* just above the horizontal level *AB*. This insures the height remaining the same, or very nearly the same, at *A* and *B*, and gives a means of measuring the height without observing the meniscus at *A* or *B*. In the bend of the tube *GK* there is compressed air, so that the pressure is always the same at the meniscus *G* and the meniscus *K*. The levels *G*, *K*, and *C* may be measured. Cold water is passed through *M*, or a mixture of ice and water placed in it, and steam is passed through *L*, thermometers at *A* and *B* indicating the tempera-

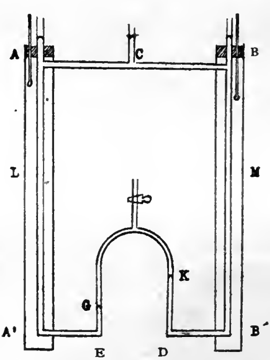


Fig. 5.

tures. Suppose that the temperature of M is t_1°C., of L is t_2°C., and of the tubes between them is t_3°C. or the temperaature of the room. Let d_1 be the density of the cold liquid in M at t_1°C.; d_2 the density of the hot liquid in L at t_3°C.; d_3 the density of the liiquid in the tubes between M and L at t_3°C.; H the vertical height of C above the level of $A'B'$; l_1 and l_2 the lengths of the columns KD and GE, and h their difference GK. Then, since the pressure of the air enclosed in the U-tube is the same at G as at K, we have

$$(1) \quad p + H\, d_2\, g - l_2\, d_3\, g = p + H\, d_1\, g - l_1\, d_3\, g,$$

where p is the atmospheric pressure and g is the acceleration due to gravity.

If β is the coefficient of cubical expansion of the liquid, referred to the volume at 0°C., d_0 its density at 0°C., V_0 the volume of a given mass m of it at 0°C., and V_1 the volume of the same mass at t_1°C., we have $V_1 = V_0\ (1 + \beta\, t_1)$. Since $V_0 = m/d_0$ and $V_1 = m/d_1$, it follows by substitution that $d_0 = d_1\ (1 + \beta\, t_1)$. This is true generally, and therefore in the present experiment. Hence

$$(2) \quad d_0 = d_1\ (1 + \beta\, t_1),$$

$$(3) \quad d_0 = d_2\ (1 + \beta\, t_2),$$

$$(4) \quad d_0 = d_3\ (1 + \beta\, t_3).$$

Substituting in (1) the values of d_1, d_2 and d_3 obtained from (2), (3) and (4) and simplifying, we get

$$\frac{H}{1 + \beta t_2} + \frac{h}{1 + \beta t_3} - \frac{H}{1 + \beta t_1} = 0 .$$

Clearing of fractions and putting the equation into the form $a\beta^2 + b\beta + c = 0$, we get by solution

$$\beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where $a = H\, t_1\, t_3 + h\, t_1\, t_2 - H\, t_2\, t_3$,

$b = H\ (t_1 + t_3) + h\ (t_1 + t_2) - H\ (t_2 + t_3)$, and

$c = + h.$

By measuring the lengths H, h and the three temperatures t_1, t_2, t_3, the value of β may at once be calculated.

(*a*) Pass cold running water through the jacket M and steam through L, keeping thermometers on the two sides. Instead of running water, it may be necessary to use a bath of ice and water in M. Wrap a cloth about the horizontal tube $A'E$, and keep this wet with cold water to make the conduction of heat to the column GE as small as possible. Note the difference in height of the menisci at G and K when the temperatures become steady. The distance h must be very accurately determined at each setting, whereas a moderately accurate reading of the height H is sufficient. Do not forget to take simultaneous readings of the heights and the temperatures. Continue the experiment long enough to be certain that the conditions are steady.

(b) Calculate the value of β. Why should this often be called the *mean zero* coefficient of the liquid between 0° and the temperature of the steam?

If the height h were measured accurately to within 0.1 mm. to what accuracy should the height H be measured?

26. COEFFICIENT OF EXPANSION OF GLASS BY WEIGHT-THERMOMETER.

The purpose of this experiment is to determine the coefficient of cubical expansion of glass by means of the weight-thermometer.

The weight-thermometer consists of a glass tube closed at one end and ending in a curved capillary at the other end. It is filled with mercury at 0°C., and the mass of the mercury measured. When later placed in a bath of higher temperature, some mercury overflows, since mercury expands more rapidly when heated than does glass. The mass of this overflow is measured.

Let M = the mass of the mercury filling the weight-thermometer at 0°C.,

V_0 = the volume of M, and hence of the weight-thermometer at 0°C.,

β, γ = the coefficients of cubical expansion respectively of mercury and glass, and

m = the mass of the mercury which overflows when the temperature is raised from 0° to t°C.

Then $V_0 (1 + \beta t)$ = the volume of the mass M at $t°$, and $V_0 (1 + \gamma t)$ = the volume of the weight-thermometer at $t°$; hence $V_0 (1 + \beta t) - V_0 (1 + \gamma t) = V_0(\beta - \gamma) t$ = the volume of the mass m at $t°$. If d_0 and d_t represent the densities of mercury at 0° and $t°$, respectively, then

$$(1) \qquad d_0 = M/V_0,$$

$$(2) \qquad d_t = m/V_0 (\beta - \gamma) t,$$

$$(3) \qquad d_0 = d_t (1 + \beta t) \qquad \text{(See Exp. 25.)}$$

By eliminating d_0 and d_t from equations (1), (2), and (3), we get

$$M(\beta - \gamma)t = m(1 + \beta t).$$

Besides containing the known masses, M and m, this equation contains the three quantities, β, γ, and t. Any two of these three quantities being known, the third will be given by the equation. In the present experiment β and t are known, and γ is to be calculated.

(*a*) Weigh the empty weight-thermometer to an accuracy of 10 mg. Then fill it with mercury. In doing so it should be held by a clamp, or suspended in a gauze jacket, and heated by a flame held in the hand, care being taken to keep from heating too rapidly and from applying the flame too long at any point of the empty bulb.

The end of the capillary dips under the surface of mercury in a porcelain dish. The mercury in this dish should first be

heated, and then the weight-thermometer gently heated until the air bubbles out through the mercury. On allowing the bulb to cool, some mercury will run into it. The process is then repeated. When considerable mercury is in the bulb, heat it until it boils vigorously, but be careful not to heat too hot that portion of the glass where there is no mercury. Keep the mercury hot in the porcelain dish, otherwise the glass is apt to crack when the cooler mercury rushes in. The tube must be completely filled with mercury to the end of the capillary, the last bubble of air being expelled. To accomplish this it will be found helpful to turn the weight-thermometer so as to give the capillary above the air-bubble an upward slant. A gentle tapping with a light splinter or pencil will then probably cause the bubble to work its way along the tube far enough to be easily expelled by further heating.

(*b*) Keeping the end of the capillary in the dish of mercury, allow the weight-thermometer to cool in the air sufficiently so that you can bear your hand on it. Then surround it with shaved ice and leave it long enough to contract as much as it will. Assume that its temperature is now 0°C. Carefully remove the dish and brush the mercury off the end of the capillary. Place a watch-glass under the end to catch the mercury as it begins to expand and flow out. Now remove the ice-bath and warm the bulb with the hand until its temperature is raised to the temperature of the room.

(*c*) Place the weight-thermometer in the boiler provided. Heat it to the boiling point of water by passing steam over it until no more mercury comes out. Read the barometer and calculate the temperature of the steam. Very carefully weigh the mercury in the watch-glass to an accuracy of 1 mg. Weigh the weight-thermometer and contained mercury to an accuracy of 10 mg.

(*d*) Using your values of M and m, and taking the coefficient of expansion of mercury as found in Exp. 25, or from

the Tables, calculate the coefficient of cubical expansion of glass.

What additional measurements would you need to have made in order to measure the room temperature with your weight-thermometer?

27. COEFFICIENT OF EXPANSION OF A LIQUID BY PYCNOMETER.

Reference.—Edser, p. 81 and p. 86.

The method consists in determining the mass of the liquid (alcohol) filling a pycnometer at each of several different temperatures, and from the data calculating the coefficient of expansion of the liquid. Four determinations should be made, at intervals of about 8°, beginning with the room temperature.

(*a*) Fill the pycnometer with alcohol and set it on a platform in a kettle of water, so that the water comes well up to the neck of the pycnometer. Hang a 50° thermometer in the bath alongside the pycnometer, and keep the bath well stirred for about five minutes. The temperature of the bath, which must be a little above that of the room, should remain constant within 0°.1 during this time, and at the end of it the alcohol will have the same temperature within 0°.1. Take the pycnometer out of the bath, wipe the outside dry, and weigh to an accuracy of 1 mg.

(*b*) Repeat with the bath at or near each of the higher temperatures selected, keeping the temperature steady for ten minutes by holding the lamp under the kettle for a few seconds occasionally. Careful trial has shown that after this treatment the temperature of the alcohol at the center of the pycnometer is about 0°.1 lower than that of the bath, and therefore the average temperature of the alcohol is the same as that of the bath to within 0°.1.

(*c*) Empty the alcohol into the bottle from which it was taken, dry the pycnometer with a jet-pump, and weigh.

(*d*) Determine the mass of the alcohol filling the pycnometer at each temperature. Plot the results, with temperatures, starting from 0°, as abscissae, and masses as ordinates. Assuming that the expansion is uniform, draw the straight line which best represents the plotted points, and from it find the mass filling the pycnometer at 0°C.

(*e*) For the time being, assume that the volume of the pycnometer remains constant. Then the ratio of the masses of alcohol filling this volume at any two temperatures (0° and 40° C., say) is equal to the inverse ratio of the volumes of a given *constant* mass at the given temperatures. (Proof?) From this calculate the *apparent* coefficient of expansion of alcohol for the given range of temperature.

(*f*) What has been found is not the *absolute* coefficient of expansion, since the pycnometer also expands. Find from your own work in Exp. 26, or from the Tables, the coefficient of cubical expansion of glass, and by applying it to the above result find the *absolute* coefficient for the alcohol.

(*g*) The formula derived in Exp. 26 for the weight-thermometer will also apply to the pycnometer as used in the present experiment. By means of this formula determine the coefficient of expansion for alcohol. Is the coefficient, as thus determined, apparent or absolute?

28. EXPANSION CURVE OF WATER.

Reference.—Duff, p. 201.

The variation of the volume of a given mass of water, as the temperature is raised by steps from the freezing point, is to be studied, taking the expansion of mercury as the temperature standard. It should be remembered that our choice of a thermometer and scale of temperatures is entirely arbi-

trary. The statement that a certain substance expands "uniformly" can mean only that it expands uniformly with the change of some property of a particular substance chosen as a standard. Taking the expansion of mercury as a standard, we wish here to determine how water changes in volume with change of temperature.

(*a*) The bulb of the water-thermometer can be filled by the aid of a reservoir-tube fitted on the end of the thermometer-stem. The reservoir is filled with water previously boiled to expel the oxygen dissolved in it. The bulb is then alternately heated to drive out the air and allowed to cool to admit water. Finally, when only a tiny air-bubble remains, this may be gotten rid of, either by jarring the tube so as to break up the bubble into smaller ones which will pass up along the stem, or by immersing the bulb in ice-water and forcing the air back into solution. If neither of these methods works, ask for assistance.

Fill the thermometer until the water stands in the stem, at 0°C., about 2 cm. above the bulb. Fasten the tube to the face of a metric scale and place the bulb in the water-bath. The bulb is first to be surrounded with shaved ice. When conditions become constant, take a reading of the height of the water meniscus and also of the mercury thermometer placed in the bath near the bulb. Melt the ice and gradually raise the temperature of the bath very carefully, at first reading the mercury and water thermometers at every degree between 0° and 8°C., then at approximately 10°, 15°, 20°, and every ten degrees thereafter as far as the water thermometer will permit.

(*b*) Determine the volume of the water in the bulb at 0° C., by weighing the bulb with the water in it and then weighing it empty and dry.

(*c*) Determine the diameter of the bore, either by direct measurement with the micrometer microscope, or by placing

in the tube a thread of mercury, measuring its length and then weighing the mercury.

(*d*) From the determination of the volume of the bulb and the diameter of the bore of the tube, calculate the volume, in cu. cm., of the water at each of the temperatures observed, making no allowance for the expansion of the glass.

On coördinate paper plot the results and draw a curve, having for abscissae the temperatures as recorded by the mercury thermometer, and for ordinates the corresponding volumes of water. In doing this, choose as large a scale for volumes as possible, so that the total change of volume will about cover the width of the sheet of paper. In order properly to show the expansion between 0° and 8°, reproduce this part of the curve, employing a magnified volume-unit. Since your observed changes of volume are only apparent changes, the volume-change of glass must be added in order to obtain the true expansion of the water. To do this, calculate what the volume-increase of the water-thermometer is between 0° and 100° C., due to the expansion of the glass, using the coefficient of cubical expansion of glass given in the Tables. At the point on the temperature axis corresponding to 100° erect an ordinate equal to this expansion. Draw a slanting line through your origin of coördinates and the upper end of this ordinate. The lengths of the ordinates between this line and the temperature axis represent the expansion of the glass for the corresponding temperatures. Then, from various points along the apparent expansion curve of water, measure, vertically upward, distances equal to the volume-increase of the glass corresponding to this temperature. Draw a smooth curve through all of the points thus plotted. This curve referred to the horizontal axis will give the true expansion of the water.

(*e*) State any conclusions that can be drawn, from an examination of the curve, in regard to the behavior of water as its temperature is raised from 0° to the highest point reached.

By the use of the curve determine the mean coefficient of expansion of water (1) between 0° and 100°, (2) between 0° and 20°, (3) between 0° and 8°.

Water Equivalent.

In most experiments, such as those involving specific heat, heat of fusion, etc., where a calorimeter and its accessories are used, it is convenient to know their water-equivalent.

By the *water-equivalent of a body* is meant the number of grams of water which would be heated (or cooled) the same number of degrees as the body for the passage into it (or out of it) of the same amount of heat. It is numerically equal to the heat capacity of the body, and is found by taking the product of the mass of the body and the specific heat of the substance of which it is made. It is called "water-equivalent" for the reason that in all calorimetric calculations the body may be replaced by this thermally equivalent mass of water. This applies directly to calorimeter cups and stirrers. As a rule, it will be necessary to consult the Tables for the values of the specific heat.

In the case of a thermometer, which is part glass and part mercury, the water-equivalent may be determined by finding the volume of the immersed part of the thermometer and then calculating the water-equivalent of this volume of mercury. This is possible since equal volumes of glass and mercury have practically the same heat capacity, and hence the thermometer may be treated as though it were made entirely of mercury. The student will find that 0.45 is approximately the factor by which the volume in cc. should be multiplied to give the water-equivalent of the thermometer.

29. SPECIFIC HEAT OF A LIQUID BY METHOD OF HEATING.

In this experiment a heating coil, composed of high resistance metal through which an electric current is passed, is immersed for a given time, first in one liquid and then in another. If the same current passes through the coil in the two cases, equal quantities of heat should be generated in equal times. Noting in each case the mass of the liquid and the rise in temperature, the two quantities of heat may be equated and the specific heat of one liquid calculated, if that of the other is known. Let m_1, m_2 be the masses of the two liquids; s_1, s_2 their specific heats; t_1, t_2 their changes in temerature; and w the water-equivalent of the calorimeter cup and accessories. Then

$$(m_1 s_1 + w\ 1)\ t_1 = (m_2 s_2 + w\ 1)\ t_2,$$

from which s_2 may be found if the rest of the quantities are known. Water, taken as a standard, will be one liquid used. *Another liquid is furnished, whose specific heat it is the object of this experiment to determine.* The method is applicable to any liquid which is not a conductor of electricity and which does not act chemically upon the material of the coil or calorimeter.

(*a*) Place the bottle, containing the second liquid, in a vessel of ice-water to cool. Weigh a quantity of ice-water in the calorimeter cup. Set up the calorimeter and immerse the heating coil, having the temperature of the water about 12° below the room temperature. Allow a few moments for the contents of the cup to come to a uniform temperature, then note the temperature, and turn on the current in the coil. Record the time when the current is started, and also the time for each rise of two or three degrees in temperature of the water until it reaches a temperature as far above that of the

room as it started below. Keep the water well stirred, and do not place the thermometer very close to the heating coil.

(*b*) Repeat (*a*), using the second liquid, instead of water, in the calorimeter cup. It will be necessary to find the water-equivalent of the calorimeter cup, stirrer, and thermometer. For this purpose see the paragraph on "Water-Equivalent."

(*c*) Plot on the same sheet of coördinate paper the results of (*a*) and (*b*), using temperatures as ordinates and times as abscissae. Erect two perpendiculars to the time axis which will include between them as wide segments of the two curves as is consistent with accuracy. From these intersections obtain the range of temperatures passed through by the water and the other liquid in equal times. Take the quantities of heat gained by the two liquids and the calorimeter in this time as equal, and form an equation from which the specific heat of the liquid may be calculated. From the result just obtained calculate what mass of the liquid will be "equivalent" to the water used in (*a*).

(*d*) If you have time, take the amount of liquid found in (*c*) to be equivalent to the water used in (*a*), and repeat (*b*). From the data obtained calculate the value of the specific heat. Why should this value be more reliable than the one found in (*c*)?

30. SPECIFIC HEAT OF A LIQUID BY METHOD OF COOLING.

Reference.—Millikan, p. 206.

This method is a comparison of the quantities of heat lost by two liquids, one of which is water, when equal volumes are allowed to cool under exactly the same conditions through a certain range of temperature. The conditions of radiation being the same for both, if a liquid of mass m_1 and specific heat s_1 cools through a certain temperature-range t in T_1 sec-

onds, and a second liquid of mass m_2 and specific heat s_2 requires T_2 seconds for the same temperature-change, then the quantities of heat lost will be proportional to the times, i. e., $Q_1/Q_2 = T_1/T_2$. If w denote the water-equivalent of the containing vessel, thermometer, and stirrer, the above relation becomes

$$\frac{(m_1 s_1 + w \, 1)\, t}{(m_2 s_2 + w \, 1)\, t} = \frac{T_1}{T_2},$$

from which the unknown specific heat can be determined.

(*a*) A large jar is used, having a wooden cover from which is suspended a smaller vessel. The space between these is made a water-jacket by putting enough water in the larger so that when the cover is put in place the space between the two vessels will be filled with water. The liquid used, turpentine, is now heated in a water-bath to about 85° and poured into a small copper cup, closed by a cork through which the thermometer and stirrer pass. The cup is then passed through the wooden cover and hangs suspended from the cork.

Make the necessary weighings on the trip-scales. Allow the turpentine to cool to about 50°, recording the time for every two degrees fall in temperature at first, and later for each one degree fall.

(*b*) Put fresh water in the jacket, and repeat (*a*) with water in the cup instead of turpentine, using as nearly as possible the same volume.

(*c*) Plot the cooling curves of turpentine and of water on coördinate paper, using temperatures as ordinates and corresponding times as abscissae. On these curves take a certain range of temperature by drawing two lines parallel to the axis of abscissae, each line cutting both curves. Make this temperature-interval as long as possible, consistent with accuracy. The intersections of these lines with the curves will give the times required. Calculate the specific heat of turpentine.

If the water had not been changed between the two sets of observations, in what way would the value for the specific heat of turpentine have been affected? What source of error still remains even if the water in the jacket is changed before the second measurement? Suggest a way to avoid this uncertainty.

31. MECHANICAL EQUIVALENT OF HEAT BY CALLENDAR'S METHOD.

The number of units of mechanical work which is equivalent to the calorie of heat is called the mechanical equivalent of heat. Most of the methods employed in determining it produce the heat by means of mechanical work done against friction. In the Callendar method a measurable amount of work done against the friction between a stationary silk belt and a revolving vessel is converted into heat in a known mass of water contained in the vessel. The apparatus consists of a brass cylindrical vessel which contains a known mass of water and whose axis is horizontal. This cylinder can be rotated at a moderate speed by hand or by motor. Over the surface of the cylinder a silk belt is wound so as to make one and a half complete turns. From the ends of this belt are suspended known masses, adjusted so as to provide a force-moment which will oppose the rotation of the vessel. An automatic adjustment for equilibrium is secured by the use of a light spring balance which acts in direct opposition to the weight at the lighter end of the belt. This spring balance contributes only a small part to the effective difference of load between the two ends of the belt, hence small errors in its reading are relatively unimportant. The masses suspended from the belt are approximately adjusted by trial to suit the friction of the belt, the final adjustment being automatically effected by the spring balance. A counter registers the number of revo-

lutions; and a bent thermometer, inserted through a central opening in the front end of the cylinder, measures the temperature.

If M is the mass at the heavier end of the belt, m the mass at the lighter end, and F the reading of the spring balance, then the force acting to oppose the rotation of the cylinder is $(M - m + F)\ g$, where g is the acceleration due to gravity. The work done in overcoming this force during one revolution of the cylinder is $2\pi r\ (M - m + F)\ g$. If, in n revolutions, the water of mass W is raised from T_1°C. to T_2°C., we have, by equating the work done and the heat generated.

$$2\pi rn\ (M - m + F)\ g = (W + w)\ (T_2 - T_1 + R)\ J,$$

where w is the water-equivalent of the cylinder and the thermometer, R is a temperature-correction to compensate for radiation, conduction and the viscosity of the water, and J is the mechanical equivalent of heat. *The purpose of the experiment is, by means of this relation, to determine the value of J from known and observed values of the other quantities involved.*

(*a*) Make a mixture of 4 or 5 liters of ice-water whose temperature is about 10° below the temperature of the room. Weigh out enough of it to fill the brass cylinder about half full. Suspend masses from the ends of the belt, so that, when the cylinder is rotated at moderate speed, the masses hang free of the fixed parts of the frame.

(*b*) Read the temperature t_1 of the water, loosen the belt so as to eliminate the friction between it and the cylinder, and perform 100 rotations of the cylinder. Record the final temperature t_2. This is done to determine the rate at which the temperature of the water is changing, due to radiation, conduction, and the viscosity of the water, just *before* the test in (*c*) is made.

(*c*) Adjust the belt and masses as they were in (*a*), read

the temperature T_1 of the water and rotate the cylinder at the same speed until the temperature has risen about 10°, and again record the temperature. Record the number of revolutions, n.

(*d*) Loosen the belt and perform 100 rotations, as in (*b*), recording the temperatures t_3 and t_4. This is done to determine the rate at which the temperature is changing, due to radiation, conduction, and the viscosity of the water, just *after* the test in (*c*) is made.

(*e*) From (*b*) it is evident that in one revolution the *fall* in temperature before the test, due to influences other than the friction of the belt, is $(t_1 - t_2)/100$; and from (*d*) the fall per revolution after the test is $(t_3 - t_4)/100$. It is reasonable to assume that, during the test in (*c*), the average loss in temperature per revolution is the mean of the two. Hence, for the n revolutions of (*c*), the loss in temperature due to radiation, conduction, and the viscosity of the water is

$$(2) \qquad R = \frac{1}{2}\frac{n}{100}(t_1 - t_2 + t_3 - t_4).$$

From the data and equations (1) and (2) determine the value of J.

Point out the principal sources of error, indicating how each affects the result.

32. MECHANICAL EQUIVALENT OF HEAT BY PULUJ'S METHOD.

This method of determining the mechanical equivalent of heat involves the measurement of the work done, through friction, in raising a given mass of mercury through a measured range of temperature. The apparatus consists of two hollow cones, one within the other, mounted on a rotating axle. Mercury is placed in the inner one and a thermometer suspended in the mercury. The outer cone is rotated and tends, through fric-

tion, to carry the inner cone with it. This is prevented by a wooden pointer attached to the inner cone. From one end of the pointer a cord is stretched parallel to a line through the axis of the cones and the axis of the rotator-wheel. This cord passes over a pulley to a scale-pan on which a mass may be placed. The deflection of the pointer may be read by means of a scale under the shorter end of the pointer. If the outer cone is rotated with constant angular velocity, the pointer will be held at a constant deflection. It is while the pointer is thus deflected that the stretched cord should be adjusted as stated above. The force-moment preventing the rotation of the inner cone is that due to the tension in the cord, and it may be measured. The value of this force-moment when multiplied by 2π gives a measure of the work done in each revolution of the outer cone.

Let M be the mass of the pan and its contents and f the friction of the pulley. Then the total force acting on the end of the pointer is $(Mg + f)$, where g is the acceleration due to gravity. If L is the lever-arm, the force-moment of this force about the axis of the cones is $L\ (Mg + f)$. Now, during each revolution of the outer cone the work done equals the force of friction times the circumference of the cone. But it can be easily shown that the force of friction times the circumference of the cone is equal to 2π times the force-moment of the friction. From the fact that the outer cone is in equilibrium under the action of friction and the tension in the cord, it follows that their force-moments must be equal; hence the work done in each revolution of the outer cone is $2\pi L\ (Mg + f)$. If W_n is the work done in n revolutions of the cones, we will have

$$W_n = 2\pi\, n\, L\ (Mg + f).$$

In making n revolutions of the cones, let us suppose that the temperature has risen from T_1 to T_2. Let the sum of the

masses of the two cones be m_1, and the specific heat of steel s_1. Let m_2 be the mass of the mercury in the inner cone, s_2 its specific heat, and w the water-equivalent of the thermometer. Then, if Q is the quantity of heat in calories generated,

$$Q = (m_1 s_1 + m_2 s_2 + w) \ [\ (T_2 - T_1) + R],$$

where R is a temperature-correction made necessary by virtue of the heat lost by the cones through radiation and conduction to surrounding objects. From these values the mechanical equivalent of heat (J) may be obtained from the relation, $J = W/Q$.

(*a*) The masses of the steel cones are given. Partially fill the inner cone with mercury, but not fuller than two centimeters below the top, and weigh. Put the inner cone, thermometer, pointer, and scale-pan in position. Rotate the outer cone and adjust the mass on the scale-pan so as to give a steady deflection of 20 degrees or more. When the pointer is deflected, the pulley should be moved so that the cord is parallel to the axis of the machine, as already explained.

(b) Read the temperature t_1 of the mercury, remove the thermometer and pointer (but not the inner cone), and perform, with the same speed as before, enough rotations of the wheel to make 200 rotations of the cones. Record the final temperature, t_2. This is done to determine the rate at which the temperature is changing, due to radiation and conduction *before* the test in (*c*) is made.

(*c*) Replace the pointer and thermometer, read the temperature, T_1, and rotate the cone steadily as before until the temperature has risen about 5°, and again record the temperature, T_2. Record the number of rotations, n, of the cones.

(*d*) Remove the pointer and thermometer and again perform 200 rotations of the cones, recording the temperatures, t_3 and t_4, as in (*b*). This enables one to determine the rate at

which the temperature is changing, due to radiation and conduction, *after* the test in (c) is made.

(e) Detach the cord from the end of the pointer and attach masses just sufficient to balance the mass of the scale-pan when the cord is hung over the pulley. Now put additional masses on one side till the system just begins to move. Call this extra mass p; then $f = pg$, where g is the acceleration due to gravity.

(f). From (b) the fall in temperature before the test, due to heat-losses during one rotation, is $(t_1 - t_2)/200$; and from (d) the fall per rotation after the test, due to the same cause, is $(t_3 - t_4)/200$. The mean of these two will fairly represent the fall per rotation during the test in (c). Hence, for the n rotations of (c), the fall in temperature due to radiation and conduction is

$$R = \frac{1}{2}\frac{n}{200}(t_1 - t_2 + t_3 - t_4).$$

Explain fully how R is given by the measurements of (b) and (d) and the last calculation. Calculate J from the equations previously given.

33. COOLING THROUGH CHANGE OF STATE.

Reference.—Duff, p. 220.

When a solution changes from the liquid to the solid state, or vice versa, the temperature at which the change occurs is, as a rule, not as sharply defined in the case of a non-crystalline substance as in the case of a crystalline one. The two types of substances act differently, also, in that supercooling often occurs in the one case and not in the other.

I. *Melting Point and Cooling Curve of Paraffine.*

(a) Take several pieces of capillary tubing, each 2 or 3 cm. long, dip the ends in melted paraffine, a non-crystalline

substance, and let them fill by capillarity. Fasten them around the bulb of a thermometer by means of a rubber band. Place the thermometer-bulb in a small corked test-tube, and immerse and heat in a water-bath, taking care not to allow any water to enter the test-tube. Note the temperature at which the paraffine melts. Remove the test-tube containing the thermometer and note the temperature at which the paraffine solidifies. Take the mean of these two as the melting point.

(*b*) Put a thermometer in a test-tube together with enough paraffine to cover the bulb when melted. Heat the test-tube in a water-bath until the thermometer registers about 70°. Remove the test-tube from the bath, clamp it in a stand, and allow the paraffine to cool slowly in air to about 38°C. Record the time for each degree or half-degree fall, or at shorter intervals when the cooling is evidently not uniform. Plot on coördinate paper, using temperatures as ordinates and times as abscissae. Explain the form of the different parts of the curve, with especial reference to rate of radiation and relative specific heat.

II. *Cooling Curve of Acetamide.*

Place a thermometer in a test-tube and surround the thermometer with crystals of acetamide, filling the test-tube about one-third full. Place the test-tube in a water-bath and heat to the temperature of boiling water. Remove the test-tube from the bath, clamp it in a stand, and either note the time for each degree or half-degree of fall, or record the temperature for every half-minute. Continue the readings until the temperature falls to about 40°C. Plot on coördinate paper, using temperatures as ordinates and times as abscissae. Explain the form of the different parts of the curve. Compare with the cooling curve of paraffine.

34. HEAT OF FUSION.

The purpose of this experiment is to determine the heat of fusion of the alloy known as Wood's fusible alloy.. Its composition is lead 25.9 parts, cadmium 7 parts, bismuth 52.4 parts, and tin 14.7 parts. The alloy is a solid at ordinary temperatures, but readily melts in hot water. The method employed will be the method of mixtures. A known mass of the metal is placed in a nickle crucible of known mass and specific heat, suspended in a copper cage of known mass and specific heat, and is heated to the temperature of boiling water. The whole is then plunged into a known quantity of cold water in a calorimeter cup and the change in temperature noted. The following changes occur wherein heat is given out: (1) The alloy cools as a liquid from the temperature of the hot water-bath down to the freezing point of the alloy, (2) the alloy changes from a liquid to a solid without change of temperature, (3) the alloy cools as a solid from its freezing point down to the final temperature of the mixture in the calorimeter, (4) meanwhile the nickle crucible and the copper cage cool from the temperature of the hot water-bath to the final temperature of the mixture. The changes wherein heat is absorbed are those accompanying the rise in temperature of the calorimeter cup and contents from the initial temperature of the cold water up to the final temperature of the mixture. From these data, if the specific heat of the metal in the solid and in the liquid state be known, the heat of fusion may be found. Let M be the mass of the alloy; m, the mass of the nickel crucible; W, the mass of the water in the calorimeter cup plus the water-equivalent of the cup, thermometer, and stirrer; s_1, the specific heat of the liquid alloy; s_2, that of the solid alloy; s_3, that of the nickel crucible; T, the melting point of the alloy; t_1, the initial temperature of the alloy and crucible; t_0, the initial temperature of the calorimeter and con-

tents; t, the final temperature; and L, the heat of fusion per gram of the alloy. Write the proper equation representing the transfer of heat in the above process, using the symbols indicated, and solve the equation for L.

(*a*) First determine the mass, in grams, of those things whose mass it is necessary to know. Place the alloy in the crucible and determine, within 3°, its melting point. To do this, stand the crucible (in a clamp provided for it) in a vessel of water. Heat the water, taking care after the water has reached a temperature of 60° that the heating be done slowly, so that the alloy will be at the same temperature as the water. The thermometer must be placed in the water and not in the alloy. The liquid alloy "wets" glass, hence some of it would be withdrawn with the thermometer and relatively large changes in mass introduced. If the thermometer were placed in the alloy and left there, there would be great danger of its breaking on the solidification of the metal. After the melting point of the alloy has been found, bring the water to the boiling point, taking care that no water gets inside the crucible. Remove the crucible, noting the time, and quickly wipe the outside; then quickly and carefully lower it into the calorimeter, right side up, with its contained alloy, letting the water run into the crucible and thus more quickly cool the alloy. Only a few seconds should elapse between the removal of the alloy from the hot water-bath and its immersion in the calorimeter. Stir the mixture continuously, noting the time and temperature when the mixture becomes uniform and starts to cool, and then again 5, 10 and 15 minutes later.

By plotting times as abscissae and temperatures as ordinates, find the temperature which the mixture should have if its temperature could be made uniform the instant the alloy enters it. The effect of radiation is thus accounted for. A little consideration will make it evident that the cooling curve

in the plot, prolonged backward to intersection with the axis of ordinates will give the correct temperature.

The specific heats of the solid and liquid alloy will be given in the Tables, or if time permits they may be found by the method of mixtures. The specific heat of nickel and copper may be found in the Tables.

Make two or three determinations, as outlined above, of the heat of fusion of Wood's alloy.

(*b*) Why is only a rough determination of the melting point of the alloy necessary? Discuss the relative accuracy with which the different masses used must be determined in order that the precision of measurement of the result may be 2 per cent. Point out the principal sources of error in the experiment.

35. HEAT OF VAPORIZATION AT BOILING POINT.

References.—Edser, p. 152; Watson's Practical Physics, p. 237.

In this experiment Kahlenberg's modification of Berthelot's apparatus[1] is used.

(*a*) Determine the boiling point of the liquid used, by carefully heating a small quantity in a test-tube or beaker by means of a water-bath.

(*b*) Weigh the calorimeter, first dry and empty, then about two-thirds full of water. Carefully dry and weigh the worm, together with the two corks which fit its ends. Set up the calorimeter with stirrer, worm, and thermometer. The boiler consists of a test-tube to which is fitted a rubber stopper. A glass tube extends through the stopper to the bottom of the test-tube; two wires also pass through the stopper, and are connected to a coil of wire which loosely surrounds a part of the glass tube. When in use the test-tube is inverted,

[1] Journal of Physical Chemistry, 1901, Vol. 5, p. 215.

enough liquid being placed in it to completely cover the coil of wire after the tube is inverted. An electric current is then sent through the coil, furnishing the heat to boil the liquid. The vapor from the boiling liquid passes downward through the glass tube and enters the worm, when the boiler is placed in position over the calorimeter.

Care should be taken to use enough liquid so that the heating coil is covered throughout the experiment. Never allow the heating current to be closed through the coil while the coil is not completely covered with liquid. Do not place the boiler over the calorimeter until the liquid boils and the vapor is issuing freely from the tube. See that the cork is removed from the free end of the worm, as the boiling must be done at atmospheric pressure, otherwise the temperature of the vapor will not be that found in (*a*).

When all is ready, note the temperature of the calorimeter, and place the boiler in its proper place so that the vapor enters the worm. Gently stir the water in the calorimeter, and read the thermometer at one-minute intervals until the temperature has risen about 5°. Turn off the current, remove the boiler, cork the ends of the worm, and continue to read the thermometer at one-minute intervals for five minutes. Remove the worm from the calorimeter, carefully dry the outside, and weigh. Pour the contents of worm and boiler into the proper bottle, and empty the calorimeter. See that the electric circuit is disconnected.

(*c*) From the series of temperatures taken determine the rise of temperature of the calorimeter, correction being made for radiation. Determine the water-equivalent of the calorimeter and contents, including the stirrer, thermometer, empty worm, and water. The necessary specific heats may be obtained from the Tables. Calculate the amount of heat gained by the calorimeter. Knowing the mass of the vapor condensed, the change in temperature of the liquid, and the spe-

cific heat of the liquid (see the Tables for the specific heat), calculate the heat transferred to the calorimeter, and determine the heat of vaporization of the liquid at its boiling point.

36. HEAT OF VAPORIZATION AT ROOM TEMPERATURE.

Reference.—Duff, p. 233.

The heat of vaporization of a liquid varies with the temperature at which vaporization takes place. In nature, vaporization takes place, for the most part, at atmospheric temperature rather than at boiling temperature. *The object of this experiment is to find the amount of heat necessary to vaporize one gram of a liquid at the room temperature.* To do this, dry air is made to bubble through the liquid, thus increasing the free surface and producing rapid evaporation. The loss of weight of the liquid gives the amount evaporated, while from the fall of temperature of the liquid and calorimeter, together with their masses and specific heats, the heat-loss can be determined and the heat of vaporization calculated.

(*a*) Carefully weigh the calorimeter cup when dry and empty, and again when containing about 100 grams of alcohol. Place the cover on the calorimeter, with the thermometer-bulb in the liquid and arranged so that dry air can be forced through the liquid by means of a small foot-bellows. Have the initial temperature of the liquid 2° or 3° above the room temperature. Pass the dry air gently through the liquid, allowing ample room for the vapor-charged air to escape, until the temperature is as much below room temperature as the initial temperature was above it. If the air is forced too rapidly through the liquid, not all of the liquid which is carried away will be vaporized. Weigh the calorimeter and remaining liquid. A 50° thermometer graduated in tenths of

a degree should be used. Wet the thermometer, with the liquid used, about as high as the depth to which it will be placed in the liquid in the calorimeter, so that as much liquid will be introduced at first as will be withdrawn later when the thermometer is removed from the calorimeter.

(*b*) Repeat (*a*) two or three times. When finished, empty and dry the calorimeter. If a liquid other than water was used, it should be poured back into its proper bottle.

(*c*) From the amount of liquid evaporated, the fall in temperature, and the water-equivalent of the thermometer, calorimeter, and liquid used, determine the heat of vaporization in (*a*) and (*b*), taking the mean as the final value. It will be necessary to assume that the heat, used up in vaporizing the liquid, all came from the calorimeter and its contents. The mean of the initial and final amounts of liquid in the calorimeter should be taken as the amount of liquid cooled.

(*d*) Point out the chief sources of error.

Give a reason why the value of the heat of vaporization of a liquid increases, when the temperature, at which the vaporization of that liquid occurs, is lowered.

Evaporation takes place from dry ice at temperatures below the freezing point. This change from solid directly to vapor is called sublimation. By what amount would you expect the heat of sublimation of ice at 0°C. to exceed the heat of vaporization of water at 0°C.?

37. FREEZING POINT OF SOLUTIONS.

References.—Watson, p. 268; Watson's Practical Physics, p. 258; Edser, p. 167.

The object of this experiment is (1) *to observe the lowering of the freezing point of water caused by dissolving table salt and sugar in it to form solutions of different concentrations.*

and (2) to determine the molecular weights of the salt and the sugar by means of this lowering.

(*a*) Using a 50° thermometer, determine the freezing point of pure water with the same apparatus as that employed in the calibration of the 100° thermometer. Then determine the freezing point of a 4 per cent solution of common salt in water. By percentage solution is here meant the number of grams of dissolved substance per 100 grams of the solution. Repeat for an 8 per cent and for a 12 per cent solution. Choose such amounts of these solutions in the three cases as will contain the same mass of the solvent, for example, 100 gm. of the 4 per cent, 104.3 gm. of the 8 per cent, and 109.1 gm. of the 12 per cent solution.

(*b*) Repeat (*a*) with aqueous solutions of sugar of 6, 12. and 18 per cent concentration.

(*c*) Tabulate the results of (*a*) and of (*b*), and for each of the six cases calculate the lowering, per gram of dissolved substance, of the freezing point of a *given* mass of water. What relation seems to hold between the change of freezing point of a given mass of water and the mass of dissolved substance?

Note the difference of freezing points for 12 per cent solutions of table salt and sugar.

(*d*) Calculate the molecular weights (M) of table salt and sugar from the relation $M = Ks/St$, where s is the number of grams of dissolved substance, S is the number of grams of the solvent, t is the depression of the freezing point, and K is a constant, whose value is 1850 for aqueous solutions.

Water in dilute solutions is thought to have the power of breaking up the molecules of some dissolved substances into ions, each ion having the same effect in lowering the freezing point as a molecule has. The result of this is that the observed lowering of the freezing point of water is three times as great in the case of some dissolved substances and twice as great in

the case of others as the value calculated by the formula. What ionizing effect, if any, has water on the table salt and sugar?

What relative lowering of freezing point of equal masses of water would you expect (*a*) if equal numbers of molecules of table salt and sugar were brought into solution, (*b*) if equal masses?

38. HEAT OF SOLUTION.

The quantity of heat absorbed in the solution of one gram of a substance in a large amount of the solvent is called its heat of solution. If heat is given out in the solution, the quantity is considered negative.

If the temperature of the salt after solution be different from that at which it was poured into the water, it will be necessary to consider its specific heat also. *According to the following method the heat of solution and the specific heat are both determined, although the former is the main object of the experiment.*

(*a*) On one of the Becker balances weigh out on pieces of dry paper two portions of salt, each of about 15 grams, to 0.01 gram. Make sure that the salt is quite dry and finely pulverized, and be careful not to leave any in the balance-pan. This amount of salt, if sodium hyposulphite be used, when dissolved in 200 grams of water will lower its temperature a little over 3°. It is best to have the cup about 3° warmer than the jacket, because the larger part of the salt dissolves in a few seconds, so that the loss of heat by radiation during this time is small; and the temperature being then reduced to about that of the jacket, there is no loss by radiation during the longer time required for the complete solution of the salt.

(*b*) Set up the calorimeter, with the jacket filled with

water at the room temperature, and the cup containing 200 grams of water about 3° warmer. Keep the stirrer moving slowly and read the temperature of the cup at intervals of one minute for about five minutes. Pour in the salt one minute after the last observation, stir rather vigorously to hasten solution, and record the final temperature.

From the series of observations, calculate the temperature of the cup at the time when the salt was poured in. The temperature of the salt at that time may be assumed to be that of the room.

(*c*) Make a similar trial with a second portion of salt, having the cup at about 40°C. Make sure that there is the proper difference between cup and jacket at the time the salt is poured in. To determine the water-equivalent of the calorimeter cup and stirrer, it will be necessary to know their masses and the specific heats of the metals of which they are made. The water-equivalent of the thermometer may be calculated from the number of cc. which it displaces when immersed in a graduate to the proper depth.

(*d*) Call the specific heat of the salt x, and its heat of solution in water y. Write for each of the cases (*b*) and (*c*) an equation involving the following quantities:

1. Heat lost by water in cup.
2. Heat lost by cup, stirrer, and thermometer.
3. Heat gained or lost by salt in changing temperature.
4. Heat absorbed during solution of salt. It will be well to assume that the salt initially is at the room temperature in the two cases.

Solve the two equations for x and y.

Why should the value of the heat of solution as obtained by this method be proportionately more accurate than that for the specific heat?

Caution:—Do not leave the solution standing in the cup. Wash it out as soon as possible.

39. HEAT OF NEUTRALIZATION.

When an aqueous solution of a strong acid is poured into an aqueous solution of a strong alkali until a neutral mixture is formed, the essential chemical reaction which occurs is the formation of water. For instance, if aqueous solutions of hydrochloric acid and sodium hydroxide are made to form a neutral mixture, although the mixture is a solution of sodium chloride (table salt), the only chemical reaction occurring is the formation of water. The heat generated is called the heat of neutralization. *The object of the present experiment is to determine the heat of neutralization corresponding to the formation of a gram-molecular weight of water.* In the case just mentioned, this will occur when 1000 gm. each of normal solutions of the acid and the alkali are mixed.

A 0.5 normal solution of each of the above compounds is furnished. By a normal solution is meant one which, in 1000 cubic centimeters of the solution, contains a mass of the compound (which is to enter into the new combination) equal in grams to its molecular weight. Thus the normal solution of sodium hydroxide is a solution which contains, in 1000 cc. of the solution, 40 gm. $(23 + 16 + 1)$ of sodium hydroxide, or 23 gm. of sodium, 16 gm. of oxygen, and 1 gm. of hydrogen. The 0.5 normal solution contains one-half as much in the same volume of solution.

It is evident that if equal volumes of these solutions be mixed, the reaction will be just completed, and the result will be a neutral solution of sodium chloride. The solutions are to be mixed in the calorimeter cup at as nearly as possible the same temperature, and the resulting rise of temperature noted. The alkali should be placed in the cup, and the acid added to it. The acid, being immediately neutralized, will then have no action on the metal of the cup.

(*a*) Measure out 100 cc. of the sodium hydroxide solution in the cup, and the same volume of the hydrochloric acid solution in the beaker. Wet the inside of the beaker with the acid solution before pouring the measured amount into it. This is to compensate for the liquid which remains in the beaker when later it is emptied. A small error is introduced by taking the second thermometer out of the beaker after reading its temperature, but this may be neglected.

If care has been taken not to handle the cup and beaker any more than is necessary, the two temperatures should be very nearly the same when ready for use. It may safely be assumed that the resulting solution of sodium chloride has risen to the final temperature from the mean of the two initial temperatures.

A direct determination of the specific heat of the sodium chloride solution is impracticable. The value, 0.987, which has been calculated by interpolation from tabulated results, may be used for this case.

Make two trials, and calculate for each the quantity of heat which would have been evolved if 1000 cc. of normal solution had been used in each case. Will this cause the formation of one gram-molecular weight of water?

(*b*) Repeat the work, if there is time, with solutions of potassium hydroxide and sulphuric acid, and compare the results with that in (*a*).

40. COEFFICIENT OF EXPANSION OF AIR AT CONSTANT PRESSURE BY FLASK METHOD.

The purpose of this experiment is to determine the coefficient of expansion of air by observing the contraction (inside of a glass flask or bulb) of a given mass of air when its temperature is lowered a measured amount.

A glass bulb with a long tubular neck closed by a stop-cock is suspended in a steam bath to bring the enclosed air to the temperature of boiling water. The stop-cock is then closed, imprisoning in the bulb a given mass of air at atmospheric pressure. The bulb is then inverted and plunged into a bath of ice-water, the stop-cock is opened, and the enclosed air is brought again to atmospheric pressure. The apparent volume of the given mass of air when contracted is found by determining the difference between the masses of ice-water filling the whole bulb and that part of the bulb not occupied by the contracted air.

Let V_2 and V_1 represent the volumes of the enclosed air at the temperature t_2 of the steam and the temperature t_1 of the ice-water, respectively. Let V'_2 be the volume of the whole bulb as determined by filling with ice-water. If α is the coefficient of expansion of air at constant pressure and γ the coefficient of cubical expansion of glass, we will have, by the definitions of α and γ,

$$(1) \quad V_2 = V_1 \left[1 + \alpha \left(t_2 - t_1 \right) \right],$$

$$(2) \quad V_2 = V'_2 \left[1 + \gamma \left(t_2 - t_1 \right) \right].$$

Solving for α, we get

$$(3) \qquad \alpha = \frac{V_2' - V_1}{V_1 \left(t_2 - t_1 \right)} + \frac{V_2'}{V_1} \gamma .$$

If the temperature t_1 of the ice-water is not 0°C., the value of α obtained from (3) will not be referred to standard initial temperature and the result cannot, therefore, be compared with the value given in the Tables.

(*a*) Thoroughly dry the bulb by rinsing out ten times or more with dry air. This is done by alternately exhausting by means of the jet-pump and admitting dry air from the room. Then hang it in the boiler with the bulb down and with the

stop-cock open. If there is any chance for the steam to enter, attach a rubber tube to the open end and place the other end of this tube where the steam can not enter. Boil the water, causing the steam to pass around the bulb until the air inside it is at the temperature of the steam. Then introduce the stop-cock thinly coated with grease, close the cock, remove from the boiler, and allow to cool. (Care is necessary here to keep the bulb air-tight and at the same time to keep the stop-cock from breaking when it is cooled.) Next place it under the surface of the ice-water, and open the stop-cock under water, allowing the water to enter but not the air to escape. After allowing time enough for the bulb to come to the temperature of the ice-water, raise the bulb so that the level of the water inside is the same as that without, thus assuring the same pressure in the enclosed air as before. Close the stop-cock, remove and dry, and then carefully weigh. In order to obtain the volume, the bulb must now be weighed full of water, and then again empty and dry. It is best to fill with ice-water and to make the weighings when it is cold, so as to get the volume at 0°C. In drying the bulb great care should be taken not to break the stop-cock by the heat. These weighings will enable you to determine V'_2, the inside volume of the whole bulb at 0°C., and V_1, the volume of the air in the bulb when under atmospheric pressure and at 0°C.

From the results of the above measurements and the coefficient of expansion of glass (see the Tables) find the coefficient of expansion of air.

(*b*) If time permits, repeat with some available gas other than air, and compare the result with that of air.

41. EXPANSION OF AIR. CONSTANT-PRESSURE AIR-THERMOMETER.

References.—Edser, p. 108; Duff, p. 187.

The object of this experiment is (1) *to determine the mean coefficient of expansion, between* 0°*C. and* 100°*C., of Air at constant pressure, by means of the constant-pressure air-thermometer; and* (2) *to test the gas-laws.* In the form of air-thermometer used dry air is contained in a glass tube graduated in cc. and closed at one end. The graduated tube is connected to an open glass tube by rubber tubing, the whole forming a "U" containing mercury. The pressure on the enclosed air can be regulated to any desired constant value by raising or lowering the open glass tube. Surrounding the graduated tube containing the air is a vessel, covered by an asbestos jacket, in which a water-bath may be placed or through which steam may be passed. The graduated tube is vertically adjustable through a sleeve in the bottom of the vessel, so that the meniscus of the mercury may be seen outside and the volume read.

Coefficient of Expansion.

(*a*) Fill the vessel with a mixture of ice and water, and, when the enclosed air has had time to come to the temperature of the bath, adjust the pressure so that it is 10 cm. less than atmospheric pressure, and read the volume.

Fill the vessel with water at 10°C., adjust the pressure to the same value as before, and again read the volume. In this way raise the temperature by steps, reading the volume of the air at 10°, 20°, 30°, 45°, 60°, 80°, taking care each time to wait long enough (three minutes or more) for the enclosed air to come to the same temperature as the bath, and each time adjusting the pressure so that it is 10 cm. less than atmospheric pressure. The mercury meniscus on the closed-tube side should always be as close to the bottom of the jacket

as will just permit of reading the meniscus. This is done so that the enclosed air will not be outside the water-bath.

Empty the vessel, place a cover over the top, and pass steam through the vessel, in at the bottom and out at the top. Use two Bunsen burners, if necessary, to obtain an abundant flow of steam. After waiting five or ten minutes for the enclosed air to reach the temperature of the steam, take another reading of the volume, the pressure conditions being the same as before.

(*b*) Make another and similar series of observations at a pressure 10 or 20 cm. above atmospheric pressure.

(*c*) Plot the observations of (*a*) and of (*b*) on the same sheet of coördinate paper, using temperatures (centigrade) as abscissæ and volumes as ordinates.

From the volume at 0°C. and the volume at 100°C., as taken from the curve, calculate, for each curve, the average apparent coefficient of expansion of the air between those temperatures. Take the mean of the two results, correct for the expansion of the glass, and obtain β, the absolute coefficient of expansion of air.

Gas Laws Tested.

(*d*) From the two curves in (*c*) find out by proportion if the volume of a gas varies directly as the absolute temperature when the mass and pressure of the gas remain constant.

By taking some particular volume and noting the temperature and pressure on each curve corresponding to that volume find out by proportion if the pressure of a gas varies directly as the absolute temperature when the mass and volume of the gas remain constant.

Similarly, by taking some particular temperature on the two curves and noting the volume and pressure on each curve corresponding to that temperature, find out if the volume of a gas varies inversely as its pressure when the mass and temperature of the gas remain constant.

42. CONSTANT-VOLUME AIR-THERMOMETER.

References.—Duff, p. 186; Millikan, p. 125.

The object of this experiment is to study the law of variation of the pressure of a given mass of enclosed air whose volume is kept constant while its temperature is changed. The air is enclosed in a glass bulb mounted on the frame used in Exps. 4 and 5. The frame is placed near a table so that the bulb may be surrounded by a water-bath, by shaved ice, or by a steam-bath, the table and an iron stand being made use of to support each bath in turn. A thermometer is placed in the bath to give its temperature. The pressure on the enclosed gas is regulated by raising or lowering the open tube, as is done in the experiment on Boyle's law. The value of this pressure may be determined from the barometer-reading and the difference in the levels of the mercury on the two sides of the frame. Each time before taking the readings, the volume of the air in the bulb is made the same by bringing the mercury meniscus to the level of the wire point inside the glass tube attached to the bulb.

Caution:— The mercury on the bulb side should always be lowered some distance before changing to a lower temperature. Be especially careful to do this before removing the steam-bath when you have taken a reading at the boiling point; otherwise, on cooling, the mercury will run into the bulb. Do not hurry in taking the readings after changing the temperature, but wait until the meniscus set at the wire-point remains stationary.

(*a*) Without any bath in the reservoir, while all is at the room-temperature, bring the mercury to the wire point and determine the difference in level of the mercury columns. Record the room-temperature, and the barometer-reading.

(*b*) After having lowered the mercury on the bulb side, surround the bulb with shaved ice, and then determine the

pressure with the meniscus at the wire point. The temperature may be taken as 0°C.

Melt the ice with warm water, and then make a series of determinations of the pressure when the water in the vessel is successively at a temperature of 10°, 20°, 30°, 45°, 60°, and 80°C. (approximately).

Remove the water-bath, substitute a steam-bath in its place, and make another determination. The temperature of the steam-bath may be found by determining the boiling point of water from the known atmospheric pressure (see Tables).

Arrange all observations in tabular form.

(*c*) Plot on coördinate paper the results of (*b*), using temperatures as abscissæ and the corresponding pressures as ordinates. Draw a smooth curve which will best represent the average position of the points of the plot.

Calculate the mean increase of pressure per degree increase in temperature from 0°C. to 100°C., and divide the result by the pressure at 0°C., using values taken from the plot. This is the temperature coefficient (β) of pressure of a gas. Write it as is a decimal and find its reciprocal. The negative of this represents what point on the absolute scale of temperatures?

(*d*) Write an equation connecting P_0, the pressure at 0°; P, the pressure at t°; t; and β.

Using this equation and the pressure obtained in (*a*), calculate the temperature of the room, thus using the apparatus as a thermometer. Compare the result with the room temperature as read from a mercury thermometer.

(*e*) Show from your results how the pressure of the gas varies with the absolute temperature, the volume remaining constant.

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43. VAPOR-PRESSURE AND VOLUME.

References.—Duff, p. 227; Millikan, p. 152.

The purpose of this experiment is to study the relation between the vapor-pressure of a saturated vapor and the volume of the vapor, when its temperature is kept constant.

An ordinary barometer tube, about a meter long and filled with mercury, is inverted in a cistern of mercury. A small amount of the liquid, whose vapor-pressure is to be studied, is introduced into the tube, rising to the top of the tube and vaporizing. By raising or lowering the tube in the cistern, the volume of the vapor can be changed. The corresponding vapor-pressure is found by determining the difference between the barometer-reading and the height of the mercury column above the mercury in the cistern.

Experience has shown that the saturated vapor above the surface of a liquid exerts a pressure which depends only on the nature and temperature of the liquid. It should follow from this, if the temperature is kept constant, that the vapor-pressure of any given liquid is independent of the volume of the vapor, as long as the vapor remains saturated.

(*a*) *Unsaturated and Saturated Vapors.* Fill a barometer tube with clean mercury to within 1 cm. of the end, and placing the thumb over the end, invert the tube slowly so as to make the large air-bubble pass up along the tube. If this is done a number of times and along different sides of the tube. most of the air-bubbles will be washed out of the tube. Now fill the tube brimming full and invert in the cistern, taking care not to allow any air to enter. Clamp the tube in a vertical position and measure the height of the mercury column above the surface of the mercury in the cistern. Compare with the reading of the laboratory barometer, and if the difference is greater than 1 cm., remove the tube and fill again more carefully.

With a medicine dropper introduce a little ether into the tube, taking care not to force any air in with it. Observe if the ether all vaporizes or not; note the height of the mercury column; and continue to add more and more ether until the mercury stops falling. Finally record the height of the mercury column and the length of the vapor-column above it. Assuming that the temperature has remained equal to room-temperature, what can you conclude about the relative pressures of saturated and unsaturated ether-vapor at the same temperature?

(*b*) *Vapor-Pressure and Volume.* Unclamp the tube and gradually lower it in the cistern, 20 cm. at a time. After waiting a minute each time for equilibrium to be established between the vapor and the liquid, read the height of the mercury column and the length of the vapor column. From the height of the mercury column and the barometer-reading the vapor-pressure can be found, while the length of the vapor-column may be taken to represent the volume of the vapor. Continue until all of the vapor is condensed. The gas which remains is air, and if present in considerable amount, its pressure will constitute an appreciable part of the measured pressure, especially in the later measurements.

(*c*) *Mixture of Vapor and Gas.* With the medicine dropper force more air into the tube and repeat the measurements of (*b*).

Remove the barometer tube from the cistern, fill again with clean mercury, introduce enough air to cause the mercury column to drop 20 cm. or so, and repeat the measurements of (*b*).

What effect does the presence of a gas in the vapor have on the results?

(*e*) *Vapor-Pressure and Nature of the Liquid.* Repeat (*a*) with alcohol, and if time permits, with water also. Does the vapor-pressure depend upon the nature of the liquid?

(*f*) Plot the results of (*a*), (*b*), (*c*), and (*d*) for ether on a

single sheet of coördinate paper, with pressures as ordinates and volumes as abscissae.

From the results and the curves determine (1) how the pressure of a vapor at a given temperature depends upon the degree of saturation, (2) how the pressure of a saturated vapor at a given temperature depends upon the volume of the vapor, and (3) whether the vapor pressure depends upon the nature of the liquid or not.

44. VAPOR-PRESSURE AND TEMPERATURE.

References.—Duff, p. 227; Millikan, p. 152.

The object of this experiment is to study the relation between the temperature and the pressure of saturated water-vapor. The method employed is that referred to in Exp. 22 as the "static" method of determining the boiling point of a liquid at different pressures. Two barometer tubes, filled with mercury, are inverted and mounted side by side in a vessel of mercury. One of the tubes contains, above the mercury, water-vapor with an excess of water present, while the other tube is left to be used as a barometer. By means of a water-bath surrounding the upper half of the tubes, the temperature of the water-vapor can be brought to any desired point. The bath is connected to a heater and the change in temperature is brought about by circulation. The pressure of the saturated water-vapor at any temperature will be the difference between the heights of the mercury columns in the two tubes.

At each temperature the pressure of a saturated vapor of a given liquid has a definite value which depends on the temperature and the nature of the liquid, but is independent of the volume of the vapor. When the temperature is raised, not only is the vapor heated and the pressure raised, but more liquid is vaporized, so there are two influences tending to increase the pressure of the vapor. The purpose of the present

experiment is to plot the curve which shows how rapidly the vapor-pressure increases as the temperature is raised, in the case of water-vapor.

(*a*) Read the heights of the mercury columns in the two tubes for ten different temperatures between room-temperature and 80°C. (approximately). To raise the temperature about 5 or 10° at a time, heat the boiler only for two or three minutes, then remove the burner, and stir the water-bath until a uniform temperature prevails. By this time the water-vapor inside the tube will have reached the temperature of the bath. In taking the temperature-readings hold the bulb of the thermometer slightly below the center of the space filled with water-vapor. The mercury-equivalent of the column of water above the mercury in the tube containing the vapor should be taken into account in estimating the pressure.

Always wait until conditions have become steady before taking readings at a new temperature.

(*b*) While the temperature is falling by radiation, take as many readings of both mercury columns (at intervals of about 5°) as time will allow.

(*c*) Plot a curve from the results of (*a*) and (*b*), with the pressures of the saturated water-vapor as ordinates and the temperatures as abscissae. Draw the curve so that it will represent the average positions of all the plotted points.

(*d*) Extend the curve back to intersect with the pressure ordinate corresponding to 0°C. Assuming that, instead of water-vapor, you are given a perfect gas whose pressure at 0°C. is the same as that of the saturated water-vapor, calculate what the pressure of the gas would be at 25°, 50°, 75°, and 100°C., its mass and volume being kept constant. For this purpose it will be convenient to use the law expressing the relation between the pressure and the absolute temperature of a given mass of a perfect gas kept at constant volume. Plot the results in a curve, and compare with the curve obtained in (*c*).

Does the pressure of saturated water-vapor increase with the temperature more or less rapidly than does the pressure of a gas kept at constant volume?

Would the results be different if the volume of the saturated vapor were kept constant? (See Exp. 43.)

Determine, from the curve in (*d*), the boiling point of water at a pressure of 50 cm.

45. HYGROMETRY.

References.—Duff, p. 240; Millikan, p. 164; Edser, p. 240.

In this experiment the dew-point and the relative and absolute humidity of the air are to be determined. The absolute humidity, d, is the density of the water-vapor present in the air, and is usually expressed in grams per cubic meter. The relative humidity is the ratio of the amount of water-vapor actually present in the air to the amount required to saturate it at the same temperature, the latter quantity being the maximum amount of water-vapor that can be held in suspension at that temperature. The relative humidity is therefore equal to d/D, where D is the maximum density of the water-vapor at the given temperature. The dew-point is the temperature at which the amount of water actually present in the air would saturate it, that is, the temperature to which the air must be lowered before the condensation of water will begin. The pressure of water-vapor is the pressure which it would exert by itself if there were no air present in the space considered. By Dalton's law this is the pressure it actually does exert when mixed with air. In a given volume the mass of vapor is proportional to the pressure, so that the relative humidity is equal to the ratio of the pressure p of the water-vapor in the air to the pressure P of saturated water-vapor at that temperature; that is, relative humidity is equal to p/P.

(I) *Regnault's Hygrometer.*

(*a*) Partially fill one of the hygrometer tubes with ether, and place a thermometer in the liquid. Force a current of air through the ether with a bicycle pump. The rapid evaporation of the liquid causes the temperature to fall. When the tube and the air immediately above it are cooled to the dew-point, moisture appears on the tube, this being detected more easily by comparison with the other tube. Note the temperature at which the dew begins to form. Allow the tube to become warm and record the temperature at which the dew disappears. Take the mean of these two as the dew-point. Make three such determinations of the dew-point.

(*b*) From the Tables find the pressure of saturated water-vapor at the dew-point and also at the temperature of the room, and calculate the relative humidity. The absolute humidity may be found by multiplying the relative humidity by D, the number of grams of saturated water-vapor in a cubic meter of air at the room-temperature (see the Tables).

II. *Wet- and Dry-bulb Hygrometer, or Auguste's Psychrometer.*

In the wet- and dry-bulb hygrometer, one bulb is covered with wicking which dips into water, so that the bulb is cooled by evaporation. Swing the hygrometer back and forth in the air so as to increase the circulation of air about the wet bulb. After the two thermometers come to constant temperatures, record the temperature t of the dry bulb, and the temperature t_1 of the wet bulb. Read the barometer. The following empirical formula may then be used:

$$p = p_1 - 0.0008\, b\, (t - t_1),$$

where p is the pressure of water-vapor present in the atmosphere and the value of which is to be found; p_1 the pressure of saturated vapor at the temperature of the wet-bulb (ob-

tained from the Tables); and b is the barometric pressure, all being expressed in millimeters of mercury. Find the pressure P of saturated water-vapor at the room-temperature from the Tables, and calculate the relative humidity. Find then the absolute humidity as in (b). From the Tables and the readings of the wet- and dry-bulb hygrometer, find the dew-point.

Compare the values obtained in I and II for the humidity and the dew-point.

46. DENSITY OF THE AIR BY THE BARODEIK.

The barodeik is an ordinary balance, having a hermetically sealed flask suspended from one scale-pan, and from the other (as a counterpoise) a glass plate so chosen as to have a surface about equal to the exterior surface of the flask. The reading of the balance-pointer on a properly graduated scale gives the density of the surrounding air.

I. *To find the difference between the barodeik reading and the true density of the air.*

(a) Set and read the barometer with great care. Read the wet- and dry-bulb hygrometer. From the Tables calculate the dew-point and also the pressure of the water-vapor in the air. Remember that "dew-point" means the temperature at which the water-vapor now in the air would be saturated, or the temperature at which the existing pressure of the water-vapor in the air would be the maximum pressure.

(b) From (a) calculate the density of the air. The mass of one cu. cm. of dry air, at 0°C., and 76 cm. pressure, is 0.001293 grams. The mass of the same volume of water-vapor, under the same conditions, is 5/8 as much. Then, if H be the barometric height, f the pressure of water-vapor, and

t the temperature, the mass of dry air in one cu. cm. of moist air is by the general gas law, $PV = RmT$,

$$M_1 = 0.001293 \frac{1}{1 + at} \frac{H - f}{76},$$

where a is the coefficient of expansion of a gas. The mass of water-vapor in the same volume is

$$M_2 = (5/8)\, 0.001293 \frac{1}{1 + at} \frac{f}{76}.$$

The sum of these two is the required density. (Deschanel, p. 400.)

(*c*) Read the barodeik. Do not touch the instrument, but, by moving the hand near the flask, set up a small vibration; then close the case, and determine the resting-point of the pointer, which is the density of the air as indicated by the instrument.

(*d*) Record the difference between the reading thus obtained and the true density found in (*b*); prefix the proper sign, so that, when added algebraically to the observed reading, it will give the true density of the air. This is the absolute correction for the scale-division to which it applies.

II. *Relative Calibration of the Barodeik Scale.*

(*a*) Read the instrument as in I (*c*). Repeat with the rider at division 2 to the right of the center of the scale, which is equivalent to adding 2 mg. to the right-hand pan of the balance; then use the rider in the corresponding position on the left-hand side.

(*b*) Repeat the readings with the rider at division 5, first on the right-hand side, then on the left-hand side.

(*c*) Using the exterior volumes of flask and plate as given on the instrument, calculate the changes in the density of the air which would produce the same effects on the instrument as the putting of the separate masses on the right pan,

and on the left pan. From these results construct a table of corrections, with the proper signs, for the different resting-points observed. Note that this is a relative calibration; that is, it gives the corrections to be applied to certain readings, as compared with one reading (namely, that when no weights were used) which is assumed correct.

(*d*) In part I the absolute correction for a certain reading was found. That reading was the same as, or not far from, the one assumed correct above, so the same absolute correction may be applied to the latter. By means of this, convert the table of relative corrections, (*c*), into a table of absolute corrections. This completes the absolute calibration of the instrument.

(*e*) Plot on coördinate paper the readings of the barodeik scale as abscissæ and the relative corrections of (*c*) as ordinates, but on a much larger scale. Show how the curve can be made to indicate absolute corrections instead of relative, by moving the horizontal axis of reference up or down by a proper amount. This converts it into an absolute calibration curve for the instrument, enabling one to find the density of the air at any time by merely reading the resting-point of the pointer.

47. COEFFICIENT OF FRICTION.

Reference.—Duff, p. 95.

When one body is caused to slide over the surface of another, the force which is brought into play to oppose the motion is called "friction." This force is parallel to the surface and opposite in direction to the motion. When the sliding body is on a level plane, the normal force is equal to the weight of the body; when on an inclined plane it is equal to the component of the body's weight normal to the plane. In either case the force of friction is equal and oppo-

site to the force necessary just to produce motion (starting friction), or to keep the body moving at constant speed (moving friction). If P is the force between the two surfaces and normal to them, and F is the force of friction, the ratio

$$\frac{F}{P} = \mu$$

is called the coefficient of starting or moving friction, as the case may be, and is usually denoted by the Greek letter μ. By measuring these forces and calculating their ratio the coefficient may be determined. A second method of determining the coefficient of friction is to vary the inclination of the plane until the body by its weight just begins to move (starting friction) or moves down the plane with constant speed (moving friction). If the angle of inclination at which this occurs is i, it can be shown that the coefficient of friction is equal to tan i.

(a) The coefficient of friction is to be found between blocks and the surface of a plane whose inclination can be varied. Take one of the blocks and weigh it. Determine the force of starting friction and also of moving friction on a level surface by applying forces to it by means of the shot-bucket and string and pulley. Calculate the coefficient of friction for the two cases.

(b) Determine the coefficient of friction for the same block and surface from the tangent of the angle obtained by varying the inclination of the plane until (1) motion commences, and (2) motion continues at constant speed.

(c) Set the plane at the angle giving constant speed down the plane, and find the force that will cause the block to move up the plane at constant speed. Calculate the coefficient of friction.

(d) Set the plane at an angle of 30° and find the force necessary to move the block up the plane at constant speed,

and then, if possible, the force necessary to make it move down the plane at constant speed. Then, by calculating the force perpendicular to the plane, find the coefficient of friction. If this process is not entirely clear, repeat with the plane at an angle of 60°. *N*

(*e*) Repeat (*a*), for starting friction, having the block "loaded" by placing a known mass on top of it. Compare the coefficient of friction found with that found in (*a*).

(*f*) Take a block having three or more surfaces of different areas but of the same smoothness, and determine (by any method) the force of friction as the block slides or is moved successively on the three surfaces.

(*g*) Take a block with surfaces of different degrees of smoothness, and determine the coefficient of starting friction for two or more sides.

(*h*) Compare the results obtained from (*a*), (*b*), (*c*), (*d*), and (*e*), stating your conclusions. What do you conclude from (*f*)? From (*g*)? Upon what does the friction between two surfaces depend?

48. CONSERVATION OF MOMENTUM. COEFFICIENT OF RESTITUTION.

Reference.—Millikan, p. 58.

In any system of bodies, which is not acted upon by outside forces and in which the several bodies may be moving with different velocities and in different directions with frequent collisions, the vector sum of the momenta remains constant. This is known as the Law of Conservation of Momentum. In our present study the number of bodies will be limited to two and velocities restricted to the same straight line, the collisions taking place centrally. Let us suppose that we have two bodies *A* and *B*, suspended by strings so that they hang in contact when at rest. Let *A* be drawn aside and then

released. At the lowest point of its swing it strikes the ball B. Let m_1 be the mass of A and u_1 its velocity just as it strikes B. Its momentum then at this instant is m_1u_1. The ball B will at once start off with a velocity v_2, say, and a momentum m_2v_2, if m_2 is its mass. The ball A may continue on with a diminished velocity, v_1; or remain at rest, if it loses all of its momentum; or it may rebound, in which case v_1 is negative. After impact the two balls will move away from each other with a relative velocity which is greater the greater their elasticity. The elasticity is taken into account in a factor called the "coefficient of restitution." The coefficient of restitution is numerically equal to the ratio of the relative velocities with which the bodies move apart after impact to that with which they approached each other before impact, i. e., it is given by the equation,

$$(1) \qquad e = \frac{V_2 - V_1}{u_1 - u_2},$$

where the velocities before impact, u_1 and u_2, and the velocities after impact, v_1 and v_2, are all in the same straight line. One or more of the velocities may be negative, or the particular value of a velocity may be zero, as in the case just outlined for the two balls where $u_2 = 0$ since the second ball was at rest before the impact. The value of e always lies between zero and unity. For "perfectly elastic" bodies $e = 1$, but for all actual bodies $e < 1$. For inelastic bodies $e = 0$. In any case, whether the bodies are elastic or inelastic, the conservation of momentum holds, i. e.,

$$(2) \qquad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$

This may be verified by determining the masses and the velocities. *The purpose of the experiment is to verify relation (2) in the simple case just outlined for two suspended balls.*

In order to find the velocity of a suspended ball as it collides, or just after collision, we make use of the fact that the

velocity is the same as that which the ball would have acquired if it had fallen the same vertical distance that it has descended in its swing before collision, or that it has risen in its swing after collision, as the case may be. Let the height be h; then, as the case may be, u or v equals $\sqrt{2gh}$ where g is the acceleration due to gravity. If the angle of the half swing is α and the length of the pendulum is l, we have,

$$(3) \qquad u, \text{ or } v, = \sqrt{2gl\,(1 - \cos\alpha)}.$$

(*a*) The numbers on the circular scale, at the bottom of the frame from which the balls are suspended, represent degrees of arc. First use the two large ivory balls, and see that they are adjusted so as to hang fully in contact and so that their centers are in line. Record the zero-reading for each ball, the other one being drawn aside. Then draw one aside through about 10° or 15° and fix it in position with a thread. Record the reading. Release it by burning the thread. Note carefully the extremity of the swing of each ball after impact. This can be done by placing a slider in the position for each ball. Several trials will be necessary to accurately determine these points. From these and the zero-readings, the arcs of the swings are found, and then by measuring the length of the cord (to the center of the ball) the velocities u_1, v_1, and v_2 can be determined. Repeat for two other starting points from which the ball is released. Determine the masses of the balls, and then calculate for the three cases the momentum of the system before and after impact.

(b) Use one large ivory ball and one small one, and repeat for one or two starting points, releasing the large ball.

(*c*) Repeat (*b*), reversing the process by releasing the small ball.

(*d*) Use two lead balls and repeat for one or two starting points, or place a layer of paraffine on each large ivory ball on adjacent sides and use them as inelastic balls.

(*e*) Calculate the coefficient of restitution for all the cases above. Does it appear to depend principally on the size of the balls or on the material?

For the same cases calculate the percentage difference in the momentum before and after impact. Does the "Conservation of Momentum" appear to hold equally well in all cases?

Calculate the percentage of loss of kinetic energy for each case. What becomes of the energy apparently lost? Is the loss greater in the more or in the less elastic bodies?

49. YOUNG'S MODULUS BY STRETCHING.

References.—Duff, pp. 119, 121; Millikan, p. 65.

Hooke's Law states that in elastic bodies, within their elastic limits, the strain or deformation produced is proportional to the change in the stress or distorting force. In particular it states that if different forces be applied to a wire, e. g., by suspending it and hanging masses from it, the amount of stretching will be (within certain limits) proportional to the applied force. For a wire of any given material the ratio of the change in stress per unit area of cross-section to the increase in length per unit length is known as Young's modulus. If P is the additional force applied to a wire of length L and cross-section a, and l is the elongation produced, the value of the ratio is

$$\frac{P}{a} \div \frac{l}{L}, \quad \text{or} \quad \frac{PL}{al}.$$

It is approximately a constant for any given material, thus verifying Hooke's law. The constant has widely varying values, however, for different materials.

To determine Young's modulus for any metal, a wire made of that metal is held vertically between two clamps. To the lower clamp C is attached the end of a rod whose upper end is loosely held in a support. From the lower end of the wire

or the clamp C, masses may be suspended and the wire stretched. Above the upper end of the rod is a micrometer screw with a divided head so that small fractions of a turn may be read. In one form of the apparatus, when the screw comes in contact with the rod an electric bell rings. If the wire is then stretched, the screw must be advanced again before ringing will occur. In this way the change in length of the wire between the two clamps is readily determined, if the pitch of the screw is known. In the other form of the apparatus a small spirit level is used, one end of which is attached to the upper end of the rod and the other end rests on the point of the micrometer screw. If the wire is stretched the screw must be turned back before the spirit level will again read as before. The amount of the stretching can thus be determined as with the other apparatus.

(a) Hang first a large enough mass on the wire to insure that it is straight at the beginning. Make a setting with the screw and read it to 0.01 mm. or less. Then increase the load by adding 500 gms. at a time (each time reading the screw), until the wire carries a load of about 3000 gm., or until the elastic limit is approached.

(b) Take the masses off, 500 gm. at a time, reading the screw each time.

(*c*) Repeat (*a*) and (*b*) at least once. Record the observations in tabulated form. Calculate and record the elongation produced by each 500 gm.

(*d*) Measure the length of the wire between the clamps, and also the diameter of the wire. In measuring the latter, apply the caliper to four or five different points along the wire.

(f) From the data for each wire calculate the mean elongation of that wire produced by a stretching force of 500 grams-weight. Expressing the force in dynes and the lengths in cm., calculate Young's modulus for each of the wires, and

compare. Should the result be independent of the diameter of the wire?

What evidence does the experiment give for the verification of Hooke's law? If there is any variation from the law, assign a reason if you can.

50. HOOKE'S LAW FOR TWISTING. COEFFICIENT OF RIGIDITY.

References.—Duff, pp. 117, 121; Millikan, p. 71.

If a cylindrical wire or rod be fixed at one end, and the free end be twisted about the axis of the wire, no change of volume will occur, but the strain in the wire is found to be one of shape or form only, a shearing strain. The tendency which the wire has to recover from this strain is called elasticity of form. For a wire of given material, length and diameter, the force-moment producing the twisting is found to be (within certain limits) proportional to the angle of twist. This statement may be deduced mathematically from Hooke's Law which states that in elastic bodies (within their elastic limits) the strain or deformation produced is proportional to the stress or distorting force. The mathematical reasoning establishing the relation between the angle of twist, the force-moment producing the twisting, and the material, length, and radius of the wire is not simple, involving integral calculus. The object of this experiment is to establish the relation experimentally.

If M is the moment of the twisting force, ϕ the angle of twist in radians, l the length of the rod, and r its radius, we may write

$$(1) \qquad \phi = \frac{2Ml}{\pi n r^4}, \quad \text{or} \quad n = \frac{2Ml}{\pi \phi r^4}.$$

In the above equation, n is constant for a given material and

is called the "coefficient of rigidity," or sometimes the "modulus of torsion."

The apparatus consists of two heavy table-clamps, one of them carrying a wheel about a half-foot in diameter. In the hub of the wheel is a socket in which the rod to be tested is centered and rigidly fastened. The other end of the rod is held in a similar socket mounted in the other clamp. A scale-pan, attached to the rim of the wheel, is for the load. Two smaller clamps, supporting graduated arcs, are placed in position at desired points along the rod. A metal pointer, clamped to the rod under each of the arcs, provides a way for determining the relative twist in the rod between the two clamps. For testing, four rods are provided, two of the same diameter but different substance, and two of the same substance but different diameter.

(*a*) Set the rod of smaller diameter in place, clamped *firmly* at both ends to prevent slipping. Place the pointers exactly 20 cm. apart and adjust the graduated arcs in such a position with reference to the pointers as to avoid errors due to parallax in making the readings. Set both pointers at the zero marks. Place a 200-gm. mass in the scale-pan and record the positions of the two pointers. The twist of the rod between them is measured by the difference between the readings of the pointers.

(*b*) Repeat (*a*), adding masses to the pan, preferably 200 gm. at a time up to about 1.5 kg., or until the "limit of elasticity" of the rod is reached. Whenever this limit is passed the rod will fail to untwist completely upon the removal of the masses in the pan. Record, in tabular form, the masses used, the corresponding angle of twist, and the increase in the angle for each 200-gm. mass added. Measure the diameter of the wheel, and the diameter of the rod, the latter with great care.

(*c*) Repeat (*a*) and (*b*) with the pointers adjusted to include lengths of 40 cm. and 80 cm. of the rod.

(*d*) Replace the rod by one of the same substance but different diameter. Measure the diameter as in (b), taking ten or more readings but using only the five smallest of them in averaging for a mean value. Repeat the measurments of (c) for a length of this rod equal to 80 cm.

(*e*) Repeat (*d*) with a rod of different substance, but having the same length and radius as that used in (*d*).

(*f*) From your results show how the angle of twist varies with the twisting moment, with the length of the rod, and with its radius.

Expressing all the quantities in equation (1) in the units of the absolute C. G. S. system, calculate the coefficient of rigidity for each of the cases above. Note if its value is dependent only on the substance, or not. Point out how the data afford a verification of Hooke's law.

If the radius of the wire be measured to an accuracy of 0.01 mm., with what accuracy should the length be measured in order that the result may be affected to the same degree by both?

51. FRICTION BRAKE. POWER SUPPLIED BY A MOTOR.

Reference.—Watson, p. 116.

The object of this experiment is to measure, by means of a friction brake, the power delivered by an electric motor, and to study the effect of altering the friction of the different parts. An electric motor, a bank of incandescent lamps arranged in parallel, and a key are connected in series with the 110-volt power-circuit. The circuit is made by pressing the key. The resistance can be decreased by introducing more lamps into the circuit. A Prony brake is used. The Prony

brake consists of a lever, one end of which is bound around a revolving shaft in such a way that the friction produced will tend to rotate the lever in the direction in which the shaft revolves. This tendency to rotate is balanced by a spring balance acting at right angles to the lever, or by the weight of masses hung from the lever. If P is the force in dynes acting on the lever to prevent rotation, and L the distance from the line of P to the center of the shaft, the power absorbed by the brake, or the work per second, will be $2\pi LnP$, where n is the number of revolutions of the shaft per second.

(*a*) Suspend a spring balance from the iron stand, and then attach it below to the lever of the brake so that, when the motor is running, the balance will oppose any tendency of the brake to rotate. Note the reading of the balance when the motor is not running. Then start the motor by gradually decreasing the resistance given by the incandescent lamps, and, with the motor running at less than full speed, tighten the belt connecting the motor to the shaft of the brake. Allow the motor to run at full speed with the belt taut, and record the number of revolutions of the shaft in three minutes, as given by the speed counter. Note the reading of the spring balance while the shaft is rotating. Take two more readings with the balance at different points along the lever.

(*b*) Tighten the screws which bind the wooden blocks of the brake against the shaft, and take measurements with three different lever arms. Note if the lamps grow brighter when the friction is increased. If so, what can be said about the dependence of the power consumed by a motor on the load? The effect may also be observed by tightening the belt connecting motor and brake-shaft.

(*c*) Calculate the power delivered by the motor for each of the six measurements. In what units is the power expressed, if the force of the balances is in dynes and the lever

arm in centimeters? Reduce the results to horse-power. If you know the method by which electrical power is computed, show how the efficiency of the motor may be calculated.

(*d*) Disconnect the friction brake, attach the spring balances to a cord, and hold or suspend them above the motor so that they will pull in parallel lines, thereby pressing the cord against half of the periphery of the motor-wheel. Allowing the motor to run at moderate speed, record the difference in the readings of the two spring balances as the cord presses against the wheel.

(*e*) Repeat (*d*) for the other pulley-wheel on the motor-shaft, exerting as nearly as possible the same tension as before. Measure the diameter of each of the wheels and see what relation exists between the friction and the radius of the wheels, the angle of contact being the same in the two cases.

52. ABSORPTION AND RADIATION.

References.—Duff, p. 253; Edser, p. 436.

It is well known that a dull black surface absorbs light more readily than a white or light-colored one. This is shown by the difficulty in illuminating a photographic dark room or a room with dark-colored hangings. The purpose of this experiment is to see whether the relations which hold for light apply also to the vibrations of longer period which are manifest to our senses only through the sensation of heat. That is, *it is proposed to study the rate of absorption of heat by black and by polished surfaces, and also the rate at which heat is radiated by these surfaces to a colder body.*

(*a*) A box lined with tin has an opening in the side in which three thermometers may be set and read from the outside of the box. The bulb of one of the thermometers is

bare, another is silvered, and the third is coated with lamp-black. All three thermometers should, initially, register the temperature of the room. Record the room-temperature. Heat water to boiling in a kettle and pour into the vessel in the box, arranging this so that the steam will not reach the thermometer-bulbs and condense on them. Record the readings of all three thermometers each minute until a steady temperature is reached. Then at an even minute remove the hot water and continue the readings till the thermometers again register the temperature of the room.

(*b*) Make a good freezing mixture in a large beaker, and place this in the box close to the thermometer-bulbs, the thermometers being equally distant from the freezing mixture. Read the temperatures each minute until they cease to fall. Remove the freezing mixture and read the thermometers as they return to room temperature.

(*c*) Plot the results of (*a*) and (*b*) on coördinate paper, using times as abscissæ and temperatures as ordinates, making the scale as large as possible. Discuss the form of the curves and the relation between the several curves. What relation exists between absorption and radiation at the highest and at the lowest temperatures reached? Connect the results with the fact that stoves are made black and the fender and knobs of the stove are nickeled.

53. RATIO OF THE TWO SPECIFIC HEATS OF AIR.

Reference.—Duff, p. 264.

The object of this experiment is to obtain the value of the ratio γ of the specific heat of air at constant pressure to its specific heat at constant volume. The method employed is a modification of that used first by Clement and Desormes. A quantity of the gas, compressed in a large flask, is momentarily

put in communication with the atmosphere to allow its pressure to fall *adiabatically* to atmospheric pressure, its temperature simultaneously falling a little. The gas, when shut off again from the atmosphere, gradually warms up to its initial temperature, causing an appreciable rise in its pressure. Let p_1 be the pressure in the compressed gas at the start, v_1 the volume of unit mass of the gas and t_1 its temperature (the same as that of the room). Let p_0, v_2, and t_2 be the corresponding values of these quantities immediately after communication between the compressed gas and the atmosphere is established. Then p_2, v_2, and t_1 will be the values of these same quantities at the end, if p_2 is the final pressure. The gas has now been in three conditions, as follows:

Condition	*Pressure*	*Vol. of* 1*gm.*	*Temperature*
I.	p_1	v_1	t_1
II.	p_0	v_2	t_2
III.	p_2	v_2	t_1

The change from I to II was adiabatic, since no time was allowed for heat to pass in or out of the gas by conduction or radiation; hence, by the law for adiabatic changes in a perfect gas,

$$(1) \qquad v_1^{\gamma} p_1 = v_2^{\gamma} p_0, \quad \text{or} \quad (v_2 / v_1)^{\gamma} = p_1 / p_0 .$$

The change, from I to III was isothermal; hence, by Boyle's law,

$$(2) \qquad v_1 p_1 = v_2 p_2, \quad \text{or} \quad (v_2 / v_1)^{\gamma} = (p_1 / p_2)^{\gamma}$$

Hence $(p_1 / p_2)^{\gamma} = (p_1 / p_0)$; or, taking the logarithm and solving for γ

$$(3) \qquad \gamma = \frac{\log p_1 - \log p_0}{\log p_1 - \log p_2}.$$

The desired ratio may be obtained, experimentally, therefore, by observing the values of the three pressures.

The apparatus consists of a large carboy provided with a

large-bore stop-cock so that the enclosed space may at pleasure be opened to, or shut off from, the atmosphere. The pressure of the enclosed air is measured by an oil manometer, whilst air can be forced into or withdrawn through another inlet. To thoroughly dry the enclosed air, some strong sulphuric acid is poured into the bottom of the carboy.

(*a*) Close the stop-cock, and with a bicycle pump introduce enough air in the carboy to give a reasonably large difference of pressure, as indicated by the manometer. Shut off connection between the carboy and the pump, and wait a few minutes until the temperature of the enclosed air is the same as that of the room, which will be when the manometer shows a steady, constant pressure. Read the manometer and the barometer. To get the value of the pressure-difference recorded by the manometer, it will be necessary to know the density of the oil used. This is posted on the apparatus.

(*b*) Open the stop-cock wide and thus connect the enclosed air with the atmosphere. Leave open only for a second, then close again. Wait some time until the temperature of the enclosed air has risen again to that of the room, as indicated by a steady, constant difference in pressure; then read the manometer.

(*c*) Using the data in (*a*) and (*b*), determine from equation (3) the value of γ for air.

(*d*) Repeat (*a*), (*b*), and (*c*) two or three times, and take the mean of the results.

Obtain from the Tables the values of the two specific heats of air, calculate their ratio, and compare with the result just found by experiment.

Point out the principal sources of error, stating how each affects the result.

Explain why the specific heat of a gas at constant pressure should be greater than its specific heat at constant volume.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	Use Table on p. 118.								
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	030	072	115	157	199	242
103	01 284	326	368	410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	036	078
105	02 119	160	202	243	284	325	366	407	449	490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	019	060	100	141	181	222	262	302
108	03 342	383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	021	060	100

INDEX.

MEASUREMENT OF PHYSICAL QUANTITIES.

Experimental work has one of two objects; either to find out *what kind* of a result follows under given conditions, or to find out the *numerical relations* between different quantities. The first class of experiments is called qualitative, the second quantitative. In the earlier days of any science qualitative experiments are numerous; when the science is more mature, the majority of the experiments are quantitative. The determination of various quantitative relations is the object of physical measurement.

In making a physical measurement, the magnitude of each quantity concerned has to be expressed in terms of some unit, and the process of measurement consists essentially in finding how many times this unit is contained in the given quantity. The distance between two points, for example, may be expressed in terms of the number of foot rules which could be laid end to end between these points.

Some quantities can thus be measured *directly*, but others can be measured only *indirectly*. Thus the density of a solid cylinder of any substance cannot be experimentally determined by finding how many times the unit of density is contained in the density of the cylinder. It would be determined usually by measuring the mass, length, and diameter of the cylinder, and from them calculating the density. The great majority of physical measurements are indirect measurements.

ERRORS.

Every measurement is subject to errors. In the simple case of measuring the distance between two points by means of a meter rod, a number of measurements usually give different

results, especially if the distance is several meters long and the measurements are made to small fractions of a millimeter. The errors introduced are due in part to—

(1) Inaccuracy of setting at the starting point.

(2) Inaccuracy of setting at intermediate points when the distance exceeds one meter.

(3) Inaccuracy in estimating the fraction of a division at the end point.

(4) Parallax in reading; that is, the line from the eye to the division line read is not perpendicular to the scale, and, where both eyes are used, the imaginary line joining the two eyes is not parallel to these division lines.

(5) The meter rod not being straight.

(6) The temperature not being that for which the meter rod was graduated.

(7) Irregular spacing of divisions.

(8) Errors in the standard from which the division of the meter rod was copied.

Besides the above there are doubtless other sources of error. It may be well here to note that blunders, such as mistakes due to mental confusion in putting down a wrong reading, or mistakes in making an addition, are not usually classed as errors.

Of the above errors, (1), (2), and (3) can be very much decreased by having fine divisions on the scale and by reading with microscopes; (4) can be made small by bringing the scale on the meter rod close to the object to be measured; (5) can be made very small by using a meter rod of special design, or, in rough work, by holding the meter rod against a straight edge; (6) can be nearly eliminated by using the meter rod only at the proper temperature, or, if its temperature and coefficient of expansion are known, by calculating a *correction* to be applied; (7) can be diminished only by a careful comparison of the lengths of the different divisions; and (8) can only be corrected for when something is known of the relative

accuracy of the standard from which the meter rod was copied. But even with the most refined methods and the most careful application of corrections, different measurements of the same distance usually give different results.

Errors due to (6), (7), and (8) may be *determinate* errors, that is, errors for which more or less accurate corrections can be calculated; whereas those due to (1), (2), and (3) are *indeterminate* errors, that is, errors for which corrections cannot be calculated. Moreover, of those errors for which corrections are not applied, some, like those due to (1), (2), and (3), will be *variable* in amount and will tend to make the value obtained sometimes too large and sometimes too small; while others, like those due to (7) and (8), when corrections for them are not applied, will be *constant* and will tend to make the value obtained always too large or always too small.

Since the average value of those variable errors which tend to make a result too large will after a considerable number of measurements be about the same as the average value of those variable errors which tend to make the result too small, the mean of a large number of measurements is usually nearly free from variable errors. In order as nearly as possible to do away with constant errors, the same quantity should be measured by as many different methods as possible. The results by the different methods will usually differ somewhat, but from them all a value can be calculated which is probably nearer the true value than is any one of the separate results.

TRUSTWORTHY AND SIGNIFICANT FIGURES.

Since all measurements are subject to errors, it is important to be able to determine how many figures of a result can be trusted.

In *direct measurements* it is usually possible to make a fairly accurate estimate of the extent to which a reading can be trusted. Thus in reading, by means of the unaided eye, the position of a fine line which crosses a meter rod, the reading

will not be in error by so much as a millimeter, but pretty surely will be in error by more than a thousandth of a millimeter. So the extent to which the reading can be trusted will lie between these limits. A person who is accustomed to estimating fractions of a small division will be rather sure of not making an error so great as the tenth of a millimeter, and he can often trust his reading to a twentieth of a millimeter.

It is convenient always to put down all the figures that can be trusted, even if some of them are ciphers. Thus the statement that a distance is 50 cm. implies that there is reason for supposing that the distance really lies between 49 cm. and 51 cm., whereas the statement that the distance is 50.00 cm. implies that there is reason for supposing that the distance really lies between 49.99 cm. and 50.01 cm. When the distance is said to be 50 cm., the second figure is the last in which any confidence can be placed; when the distance is said to be 50.00 cm., the fourth figure is the last in which any confidence can be placed. This fact is indicated by saying that in the first case the quantity can be trusted to the second significant figure, and in the other case to the fourth. By the number of significant figures in a quantity is meant the number of trustworthy figures, counting from the left, irrespective of the decimal point; thus there are two significant figures in 0.000026. If a distance is about 50000 km. and the third significant figure is the last in which any confidence can be placed, this fact may be indicated by saying that the distance is 50.0×10^3 km.

In *indirect measurements* the result is usually calculated by some formula. To find out how many figures should be kept in the result consider the following two cases:—

1. If the result is the algebraic sum of several quantities, such as 214.3, 36.41, and 0.506, it is seen that in the sum 251.216, no figure beyond that in the first decimal place can be trusted, because, in the quantity which has the fewest trustworthy decimal places, namely 214.3, no figure beyond the 3 can be trusted, otherwise it would have been expressed. So

the sum will not be written 251.216, but will be written 251.2. This suggests the following rule:—

RULE I. *In sums and differences no more decimal places should be retained than can be trusted in the quantity having fewest trustworthy decimal places.*

2. If the result is the product of two quantities, such as 314.428 and 32.6, then the product cannot be trusted to more significant figures than appear in the quantity having fewest trustworthy figures, irrespective of the decimal place. To make this clear, consider the following products:—

$$
\begin{aligned}
314.428 \times 32.5 &= 10218.9100 \\
314.428 \times 32.6 &= 10250.3528 \\
314.428 \times 32.7 &= 10281.7956 \\
314. \quad\;\; \times 32.6 &= 10236.4
\end{aligned}
$$

It is seen from the first, second and third products that if the quantity which is supposed to be 32.6 is really 32.5 or 32.7, then after the first three significant figures the true value of the product differs materially from the value obtained. The second and fourth of the above products show that if more than three significant figures cannot be trusted in one of two quantities which are to be multiplied, it is not worth while to use more than three, or at most four, significant figures of the other. The product in this case would be written 1.02×10^4, or at most 1.024×10^4. These facts suggest the following rule:

RULE II. *In products and quotients no more significant figures should be kept than can be trusted in the quantity having fewest trustworthy figures.*

Until the final result is reached, it is often worth while to keep one more figure than the above rules indicate.

For logarithms a safe rule is the following:—

RULE III. *When any of the quantities which are to be multiplied or divided can be trusted no closer than* 0.01 *of one per cent., use a five-place table; when any of them can be trusted*

no closer than 0.1 *of one per cent., use a four-place table; and when any of them can be trusted no closer than* 1 *per cent., use a slide rule.*

REQUIRED ACCURACY OF MEASUREMENT.

From *Rule I* it will be seen that if a small quantity is to be added to a large one, the percentage accuracy of the measurement of the small quantity need not be so great as that of the large one. Thus if $H = a + b$, and if a is about 100 cm. and b about 1 cm., a 1 per cent. error in a will produce in H no greater effect than a 100 per cent. error in b. When quantities are to be added or subtracted, they should be measured to the same number of decimal places.

From *Rule II* it will be seen that if a small quantity and a large one are to be multiplied, the percentage accuracy of the measurement of the small quantity should be at least as great as that of the large one. Thus if $H = ab$, a 1 per cent. error in a will produce in H the same effect as a 1 per cent. error in b. So that if a is about 100 cm., and b is about 1 cm., and if b cannot be trusted closer than 0.01 cm., there is no gain in accuracy by measuring a much closer than 1 cm. When quantities are to be multiplied or divided they should be measured to within the same fractional part of themselves, for example, all of them within 1 per cent. and none of them much closer, or all of them within 0.01 of one per cent and none of them much closer.

The last statement needs modification in the case of a power. If the value found for a quantity a is 1 per cent too large, that is, is $1.01a$, then the value that will be obtained for a^2 is $1.0201a$, which is about 2 per cent too large, and the value obtained for a^3 *is* $1.030301a$, which is about 3 per cent. too large. In general, if the value found for a is k per cent too large, the value that will be obtained for a^n will be nk per cent. too large. So that a quantity which is to be squared, cubed, or raised to

some higher power should be measured with more care than if it entered the formula only to the first power.

It is evident, then, that a preliminary study of the required accuracy of measurement will not only save much time, by pointing out those quantities which need to be measured with only a rough accuracy, but will also serve to determine those quantities, usually the smallest, in the measurement of which great care must be taken and sensitive instruments used.

(*Largely reproduced from Ferry & Jones.*)

TABLE I.

USEFUL NUMERICAL RELATIONS.

Mensuration.

Circle: circumference = $2\pi r$; area = πr^2.
Sphere: area = $4\pi r^2$; volume = $4/3\pi r^3$.
Cylinder: volume = $\pi r^2 l$.

Length.

1 centimeter (cm.) = 0.3937 in.	1 inch (in.) = 2.540 cm.
1 meter (m.) = 3.281 ft.	1 foot (ft.) = 0.3048 m.
1 kilometer (km.) = 0.6214 mi.	1 mile (mi.) = 1.609 km.
1 micron (μ) = 0.001 mm.	1 mil = 0.001 in.
= 0.00394 in.	= 0.00254 cm.

Area.

1 sq. cm. = 0.1550 sq. in.	1 sq. in. = 6.451 sq. cm.
1 sq. m. = 10.674 sq. ft.	1 sq. ft. = 0.09290 sq. m.

Volume.

1 cc. = 0.06103 cu. in.	1 cu. in. = 16.386 cc.
1 cu. m. = 35.317 cu. ft.	1 cu. ft. = 0.02832 cu. m.
1 liter (1000 cc.) = 1.7608 pints.	1 quart = 1.1359 liters.

Mass.

1 gram (gm.) = 15.43 gr.	1 grain (gr.) = 0.06480 gm.
1 kilogram (kg.) = 2.2046 lb.	1 pound (lb.) = 0.45359 kg.

Density.

1 gm. per cc. = 62.425 lb. per cu. ft.
1 lb. per cu. ft. = 0.01602 gm. per cc.

Force.

1 gram's weight (gm. wt.) = 980.6 dynes (g_0 = 980.6 cm./sec.^{2}.)
1 pound's weight (lb. wt.) = 0.4448 megadynes (g_0 = 980.6.)

(The "gm. wt.' is here defined as the force of gravity acting on a gram of matter at sea-level and 45° latitude. The "lb. wt." is similarly defined.)

TABLE II.

USEFUL NUMERICAL RELATIONS.

Pressure and Stress.

1 cm. of mercury at 0°C.
= 13.596 gm. wt. per sq. cm.
= 0.19338 lb. wt. per sq. in.

1 in. of mercury at 0°C.
= 34.533 gm. wt. per sq. cm.
= 0.49118 lb. wt. per sq. in.

Work and Energy.

1 kilogram-meter (kg. m.)	= 7.233 ft. lb.
1 foot-pound (ft. lb.)	= 0.13826 kg. m.
1 joule	= 10^7 ergs.
1 foot-pound	= 1.3557×10^7 ergs. ($g_0 = 980.6$ cm./sec.^{2}.)
1 foot-pound	= 1.3557 joules ($g_0 = 980.6$.)
1 joule	= 0.7376 ft. lb ($g_0 = 980.6$.)

Power (or Activity).

1 horse-power (H. P.)	= 33000 ft. lb. per min.
1 watt = 1 joule per sec.	= 10^7 ergs per sec.
1 horse-power	= 745.64 watts ($g_0 = 980.6$ cm./sec.^{2})
1 watt	= 44.28 ft. lb. per min. ($g_0 = 980.6$)

Thermometric Scales.

$C = 5/9(F - 32)$ | $F = 9/5C + 32$.

($C \equiv$ centigrade temperature; $F \equiv$ Fahrenheit temperature.)

Mechanical Equivalent.

1 gm.-calorie $= 4.187 \times 10^7$ ergs.
= 0.4269 kg. m. ($g_0 = 980.6$ cm./sec.^{2}.)
= 3.088 ft. lb. ($g_0 = 980.6$.)

TABLE III.

DENSITY OF DRY AIR.

(Values are given in gms. per cc.)

Temp. C.	Barometric Pressure (Centimeters of Mercury)					
	72	73	74	75	76	77
0°	.001225	.001242	.001259	.001276	.001293	.001310
1	220	237	254	271	288	305
2	216	233	250	267	283	300
3	212	228	245	262	279	296
4	207	224	241	257	274	290
5°	.001203	.001219	.001236	.001253	.001270	.001286
6	198	215	232	248	265	282
7	194	211	227	244	260	277
8	190	206	223	239	256	272
9	186	202	219	235	251	268
10°	.001181	.001198	.001214	.001231	.001247	.001263
11	177	194	210	226	243	259
12	173	189	206	222	238	255
13	169	185	202	218	234	250
14	165	181	197	214	230	246
15°	.001161	.001177	.001193	.001209	.001225	.001242
16	157	173	189	205	221	237
17	153	169	185	201	217	233
18	149	165	181	197	213	229
19	145	161	177	193	209	224
20°	.001141	.001157	.001173	.001189	.001204	.001220
21	137	153	169	185	200	216
22	133	149	165	181	196	212
23	130	145	161	177	192	208
24	126	141	157	173	188	204
25°	.001122	.001138	.001153	.001169	.001184	.001200
26	118	134	149	165	180	196
27	114	130	145	161	176	192
28	110	126	142	157	172	188
29	107	122	138	153	169	184
30°	.001103	.001119	.001134	.001149	.001165	.001180

Corrections for Moisture in the Atmosphere.

Dew-point.	Subtract.	Dew-point.	Subtract.	Dew-point.	Subtract.
—10°	.000001	+ 2°	.000003	+14°	.000007
— 8	2	+ 4	4	+16	8
— 6	2	+ 6	4	+18	9
— 4	2	+ 8	5	+20	.000010
— 2	3	+10	6	+24	13
0	3	+12	6	+28	16

TABLE IV.

DENSITIES AND THERMAL PROPERTIES OF GASES.

The densities are given at 0°C. and 76 cm. pressure, and the specific heats at ordinary temperatures. The coefficients of cubical expansion (at constant pressure) of the gases listed below are not given in this Table; they are about the same for all the permanent gases, being approximately 1/273 or 0.003663, if referred in each case to the volume of the gas at 0°C.

Gas or Vapor.	Formula	Density (gms. per cc.)	Molecular Weight	$C_p \div C_v$	C_p (cals. per gm.)
Air	—	0.001293	—	1.41	0.237
Ammonia	NH_3	0.000770	17.06	1.33	.530
Carbon dioxide	CO_2	0.001974	44.00	1.29	.203
Carbon monoxide	CO	0.001234	28.00	1.40	.243
Chlorine	Cl_2	0.003133	70.90	1.32	.124
Hydrochloric acid	HCl	0.001616	36.46	1.40	.194
Hydrogen	H_2	0.0000896	2.016	1.41	3.410
Hydrogen sulphide	H_2S	0.001476	34.08	1.34	.245
Nitrogen, pure	N_2	0.001254	28.08	1.41	.244
Nitrogen, atmospheric	—	0.001257	—	—	—
Oxygen	O_2	0.001430	32.00	1.41	.218
Steam (100°C.)	H_2O	0.000581	18.02	1.28	.421
Sulphur dioxide	SO_2	0.002785	64.06	1.26	.154

TABLE V.

DENSITY AND SPECIFIC VOLUME OF WATER.

Temp. C.	Density (gms. per cc.)	Specific Volume (cc. per gm.)	Temp. C.	Density (gms. per cc.)	Specific Volume (cc. per gm.)
0°	0.999868	1.000132	20°	0.99823	1.00177
1	927	073	25	777	294
2	968	032	30	567	435
3	992	008	35	406	598
3.98	1.000000	000	40	224	782
5	.999992	008	50	.98807	1.01207
6	968	032	60	324	705
7	929	071	70	.97781	1.02270
8	876	124	80	183	902
9	808	192	90	.96534	1.03590
10	727	273	100	.95838	1.04343
15	126	874	102	693	501

TABLE VI.

DENSITIES AND THERMAL PROPERTIES OF LIQUIDS.

The values given in this Table are mostly for pure specimens of the liquids listed. The student should not expect the properties of the average laboratory specimen to correspond exactly in value with them. With a few exceptions the densities are given for ordinary atmospheric temperature and pressure. The specific heats and coefficients of expansion are in most cases the average values between 0° and 100°C. The boiling points are given for atmospheric pressure, and the heats of vaporization are given at these boiling points.

Liquid	Density gms. per cc.	Specific Heat calories per gm. per deg.	Coefficient of Cubical Expansion per degree C.	Boiling Point degrees C.	Heat of Vaporization cals. per gm.
Alcohol (ethyl)	0.794	.68	.00111	78	205*
" (methyl)	.796	.60	.00143	66	262†
Benzene	.880	.42	.00123	80	93.2
Carbon bisulphide	1.29	.24	.00120	46.6	84
Ether	.74 (0°)	.55	.00162	35	90
Glycerine	1.26	.576	.000534		
Hydrochloric acid	1.27	.75	.000455	110	
Mercury	13.596 (0°)	.033	.0001815	357	67
Olive oil	.918	.47	.000721		
Nitric acid	1.56	.66	.00125	86	115
Sea-water	1.025	.938			
Sulphuric acid	1.85	.33	.00056	338	122
Turpentine	.873	.47	.00105	159	70
Water	See Tab. V.	1.00	See Tab. V.	100	537

* The heat of vaporization of ethyl alcohol at 0°C. is 236.5.
† The heat of vaporization of methyl alcohol at 0°C. is 289.2.

TABLE VII.

DENSITIES AND THERMAL PROPERTIES OF SOLIDS.

The values given in this Table are mostly for pure specimens of the substances listed. The student should not expect the properties of the average laboratory specimen to correspond exactly in value with them. As a rule the densities are given for ordinary atmospheric temperature and pressure. The specific heats and coefficients of expansion are in most cases the average values between 0° and 100°C. The melting points and heats of fusion are given for atmospheric pressure.

Solid.	Density. gms per cc.	Specific Heat. cals. per gm.	Coefficient of Linear Expansion. per degree C.	Average Melting Point. degrees C	Heat of Fusion. cals. per gm.
Aluminum	2.70	0.219	.0000231	658	
Brass, cast	8.44	.092	.0000188		
" , drawn	8.70	.092	.0000193		
Copper	8.92	.094	.0000172	1090	43.0
German-silver	8.62	.0946	.000018	860	
Glass, crown	2.6	.161	.0000090		
" , flint	3.9	.117	.0000079		
Gold	19.3	.0316	.0000144	1065	
Hyposul. of soda	1.72	.445		48	
Ice	.918	.502	.000051	0	80.
Iron, cast	7.4	.113	.0000106	1100	23-33
" , wrought	7.8	.115	.000012	1600	
Lead	11.3	.0315	.000029	326	5.4
Mercury		.0319		—39	2.8
Nickel	8.90	.109	.0000128	1480	4.6
Paraffin, wax	.90	.560	.000008-23	52	35.1
" , liquid		.710			
Platinum	21.50	.0324	.0000090	1760	27.2
Rubber, hard	1.22	.331	.000064		
Silver	10.53	.056	.0000193	960	21.1
Sodium chloride	2.17	.214	.000040	800	
Steel	7.8	.118	.000011	1375	
Wood's alloy, solid	9.78	.0352		75.5	8.40
" " , liquid		.0426			

TABLE VIII.

SURFACE TENSION OF PURE WATER IN CONTACT WITH AIR.

Temp. C.	Tension (dynes per cm.)	Temp. C.	Tension (dynes per cm.)	Temp. C.	Tension (dynes per cm.)
0°	75.5	30°	71.0	60°	66.0
5	74.8	35	70.3	65	65.1
10	74.0	40	69.5	70	64.2
15	73.3	45	68.6	80	62.3
20	72.5	50	67.8	100	56.0
25	71.8	55	66.9	Crit. Temp.	0.0

TABLE IX.

SURFACE TENSIONS OF SOME LIQUIDS IN CONTACT WITH AIR.

		Dynes per cm.			Dynes per cm.
Alcohol (ethyl)	at 20°	22-24	Mercury	at 20°	470-500
Alcohol (methyl)	at 20°	22-24	Olive oil	at 20°	32 - 36
Benzene	at 15°	28-30	Petroleum	at 20°	24 - 26
Glycerine	at 18°	63-65	Water (pure)	at 20°	72 - 74

TABLE X.

VISCOSITY OF WATER.

Temp. C.	Coeff. of Visc. (C. G. S. Units)	Temp. C.	Coeff. of Visc. (C. G. S. Units)	Temp. C.	Coeff. of Visc. (C. G. S. Units)
0°	0.0178	25°	0.0089	60°	0.0047
5	.0151	30	.0080	70	.0041
10	.0131	35	.0072	80	.0036
15	.0113	40	.0066	90	.0032
20	.0100	50	.0055	100	.0028

TABLE XI.

VISCOSITY OF AQUEOUS SOLUTIONS OF SUGAR.

% Sugar	Coeff. at 20°C. (C. G. S. Units.)	Coeff. at 30°C. (C. G. S. Units.)
0	0.0100	0.0080
5	.0117	.0089
10	.0132	.0104
20	.0191	.0145
40	.0600	.0423

TABLE XII.

COEFFICIENTS OF FRICTION.

Substances.	Static Coefficient.	Kinetic Coefficient.
Metals on metals (dry)	from 0.2 to 0.4	from 0.18 to 0.35
" " " (wet)	" 0.15 " 0.3	" 0.14 " 0.28
" " " (oiled)	" 0.15 " 0.2	" 0.14 " 0.18
Wood on wood (dry)		
(a) direction of fiber	" 0.5 " 0.7	" 0.2 " 0.3
(b) normal to fiber	" 0.4 " 0.6	" 0.18 " 0.3
Leather belt on wood pulley	" 0.45 " 0.6	" 0.3 " 0.5
" " " iron "	" 0.25 " 0.35	" 0.2 " 0.3

TABLE XIII.

ELASTIC CONSTANTS OF SOLIDS.

(Approximate Values.)

Substance.	Bulk-Modulus. (C. G. S. Units.)	Simple Rigidity. (C. G. S. Units.)	Young's Modulus. (C. G. S. Units.)
Aluminum	5.5×10^{11}	2.5×10^{11}	6.5×10^{11}
Brass, drawn	10.8 x "	3.7 x "	10.8 x "
Copper	16.8 x "	4.5 x "	12.3 x "
German-Silver	———	4.5 x "	12.8 x "
Glass	———	2.4 x "	7.0 x "
Iron, wrought	14.6 x "	7.7 x "	19.6 x "
Steel	18.4 x "	8.2 x "	21.4 x "

TABLE XIV.

(a) BOILING POINT OF WATER AT DIFFERENT BAROMETRIC PRESSURES.

(b) VAPOR-TENSION OF SATURATED WATER-VAPOR.

(This table may be used either (*a*) to find the boiling point t of water under the barometric pressure P, or (*b*) to find the vapor-tension P of water-vapor saturated at the temperature t, the dew-point.)

$t°$ Cent.	P cm.	D gm.|cc.	$t°$ Cent.	P cm.	D gm.|cc.	$t°$ Cent.	P cm.	D gm.|cc.
—10°	.22	2.3×10^{-6}	30°	3.15	30.1×10^{-6}	88°.5	49.62	
— 9	23	2.5x "	35	4.18	39.3x "	89	50.58	
— 8	.25	2.7x "	40	5.49	50.9x "	89.5	51.55	
— 7	.27	2.9x "	45	7.14	65.3x "	90	52.54	428.4×10^{-6}
— 6	.29	3.2x "	50	9.20	83.0x "	90.5	53.55	
— 5	.32	3.4x "	55	11.75	104.6x "	91	54.57	
— 4	.34	3.7x "	60	14.88	130.7x "	91.5	55.61	
— 3	.37	4.0x "	65	18.70	162.1x "	92	56.67	
— 2	.39	4.2x "	70	23.31	199.5x "	92.5	57.74	
— 1	.42	4.5x "	71	24.36		93	58.83	
0	.46	4.9x "	72	25.43		93.5	59.96	
1	.49	5.2x "	73	26.54		94	61.06	
2	.53	5.6x "	74	27.69		94.5	62.20	
3	.57	6.0x "	75	28.88	243.7 "	95	63.36	511.1
4	.61	6.4x "	75.5	29.49		95.5	64.54	
5	.65	6.8x "	76	30.11		96	65.74	
6	.70	7.3x "	76.5	30.74		96.5	66.95	
7	.75	7.7x "	77	31.38		97	68.18	
8	.80	8.2x "	77.5	32.04		97.5	69.42	
9	.85	8.7x "	78	32.71		98	70.71	
10	.91	9.3x "	78.5	33.38		98.2	71.23	
11	.98	10.0x "	79	34.07		98.4	71.74	
12	1.04	10.6x "	79.5	34.77		98.6	72.26	
13	1.11	11.2x "	80	35.49	295.9 "	98.8	72.79	
14	1.19	12.0x "	80.5	36.21		99	73.32	
15	1.27	12.8x "	81	36.95		99.2	73.85	
16	1.35	13.5x "	81.5	37.70		99.4	74.38	
17	1.44	14.4x "	82	38.46		99.6	74.92	
18	1.53	15.2x "	82.5	39.24		99.8	75.47	
19	1.63	16.2x "	83	40.03		100	76.00	606.2 "
20	1.74	17.2x "	83.5	40.83		100.2	76.55	
21	1.85	18.2x "	84	41.65		100.4	77.10	
22	1.96	19.3x "	84.5	42.47		100.6	77.65	
23	2.09	20.4x "	85	43.32	357.1 "	100.8	78.21	
24	2.22	21.6x "	85.5	44.17		101	78.77	
25	2.35	22.9x "	86	45.05		102	81.60	
26	2.50	24.2x "	86.5	45.93		103	84.53	
27	2.65	25.6x "	87	46.83		105	90.64	715.4 "
28	2.81	27.0x "	87.5	47.74		107	97.11	
29	2.97	28.5x "	88	48.68		110	107.54	840.1 "

TABLE XV.

THE WET- AND DRY-BULB HYGROMETER. DEW-POINT.

This Table gives the vapor-pressure, in mercurial centimeters, of the water-vapor in the atmosphere corresponding to the dry-bulb reading t°C. (first column) and the difference (first row) between the dry-bulb and wet-bulb readings of the hygrometer. Having obtained from this Table the value of the vapor-pressure for a given case, the dew-point can be found by consulting Table XIV. The data given below are calculated for a barometric pressure equal to 76 cm.

t°C.	Difference between Dry-bulb and Wet-bulb Readings.										
	0°	*1°*	*2°*	*3°*	*4°*	*5°*	*6°*	*7°*	*8°*	*9°*	*10°*
	cm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.
10°	.92	.81	.70	.60	.50	.40	.31	.22	.13	....	....
11	.98	.87	.76	.65	.55	.45	.35	.26	.17	....	....
12	1.05	.93	.82	.71	.60	.50	.40	.30	.21	.12	.03
13	1.12	1.00	.89	.76	.65	.55	.45	.35	.25	.16	.07
14	1.19	1.07	.94	.83	.71	.61	.50	.40	.30	.20	.11
15	1.27	1.14	1.01	.90	.78	.66	.55	.45	.34	.25	.15
16	1.35	1.22	1.09	.97	.84	.73	.60	.50	.40	.30	.19
17	1.44	1.30	1.17	1.04	.91	.80	.67	.56	.45	.35	.24
18	1.54	1.39	1.25	1.12	.99	.86	.74	.63	.51	.40	.30
19	1.63	1.49	1.34	1.20	1.07	.94	.81	.69	.57	.46	.35
20	1.74	1.59	1.43	1.29	1.15	1.02	.88	.76	.64	.52	.41
21	1.85	1.69	1.53	1.38	1.24	1.10	.96	.84	.71	.59	.47
22	1.97	1.80	1.64	1.48	1.33	1.19	1.05	.91	.78	.66	.54
23	2.09	1.92	1.75	1.59	1.43	1.28	1.13	1.00	.86	.73	.61
24	2.22	2.04	1.86	1.70	1.53	1.38	1.23	1.09	.94	.81	.68
25	2.35	2.17	1.99	1.81	1.64	1.48	1.33	1.18	1.03	.90	.76
26	2.50	2.31	2.11	1.94	1.76	1.59	1.43	1.28	1.13	.98	.84
27	2.65	2.45	2.25	2.07	1.88	1.71	1.54	1.38	1.23	1.08	.93
28	2.81	2.60	2.40	2.20	2.01	1.83	1.66	1.49	1.33	1.18	1.02
29	2.98	2.76	2.55	2.35	2.15	1.96	1.78	1.61	1.44	1.28	1.12
30	3.15	2.93	2.71	2.50	2.29	2.10	1.91	1.73	1.55	1.39	1.23

TABLE XVI.

Miscellaneous.

(1). *Heat of Neutralization.*

Any strong acid with any strong alkali evolves (+) about 761 calories for every gm. of water formed.

(2.) *Heat of Solution.*

For Calcium oxide (CaO),	+ 327	cals. per gm.
" Sodium chloride (NaCl),	— 21	" " "
" " hydroxide (NaOH),	+ 248	" " "
" hyposulphite ($Na_2S_2O_3 + 5H_2O$),	— 44.8	" " "

(3). *Lowering of Freezing Point of Water.*

For dilute aqueous solutions of any salt the lowering is proportional to the mass of salt dissolved in the same mass of water. If ionization does not occur, each gram-molecular weight of the salt in 1000 gm. of water will lower the freezing point 1.86°. If ionization occurs, however, the lowering is increased two, three, or more times, depending upon the number of separate parts or ions into which the molecule of the salt is divided by the water.

TABLE XVII.

Natural Sines.

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
0°	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

TABLE XVII. (Cont'd.)

Natural Sines.

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
45°	7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	4	4
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	1
85	9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	9998	9999	9999	9999	9999	1.000 nearly	1.000 nearly	1.000 nearly	1.000 nearly	1.000 nearly	0	0	0	0	0

TABLE XVIII.

Natural Tangents.

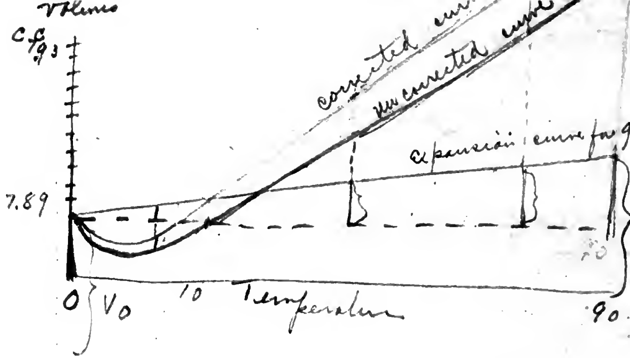
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
0°	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	14
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	.0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	.0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	.1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	.1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	.1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	.1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	.1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	.2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	.2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	.2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	.2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	.3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	.3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	.3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	6	10	13	17
20	.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	.3839	3859	3879	3899	3919	3939	3959	3978	4000	4020	3	7	10	13	17
22	.4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	.4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	.4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	10	14	18
25	.4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	.4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	.5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	.5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	.5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	.6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	.6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	.6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	.6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	.7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	.7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	.7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	10	14	19	24
39	.8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	.8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	.9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	.9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

TABLE XVIII. (Cont'd.)

Natural Tangents.

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
45°	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1.0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1.0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1.1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	26	33
49	1.1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1.2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1.2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	23	31	39
53	1.3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1.3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1.4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1.5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1.6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1.6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1.7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1.8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1.8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1.9626	9711	9797	9883	9970	$\bar{0}057$	$\bar{0}145$	$\bar{0}233$	$\bar{0}323$	$\bar{0}413$	15	29	44	58	73
64	2.0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2.1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2.2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	74	92
67	2.3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2.4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2.6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	118
70	2.7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	130
71	2.9042	9208	9375	9544	9714	9887	$\bar{0}061$	$\bar{0}237$	$\bar{0}415$	$\bar{0}595$	29	58	87	115	144
72	3.0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3.2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3.4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	82	122	162	203
75	3.7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	94	139	186	232
76	4.0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	214	267
77	4.3315	3662	4015	4374	4737	5107	5483	5864	6252	6646	62	124	186	248	310
78	4.7046	7453	7867	8288	8716	9152	9594	$\bar{0}045$	$\bar{0}504$	$\bar{0}970$	73	146	219	292	365
79	5.1446	1929	2422	2924	3435	3955	4486	5026	5578	6140	87	175	262	350	437
80	5.6713	7297	7894	8502	9124	9758	$\bar{0}405$	$\bar{1}066$	$\bar{1}742$	$\bar{2}432$					
81	6.3138	3859	4596	5350	6122	6912	7920	8548	9395	$\bar{0}264$					
82	7.1154	2066	3002	3962	4947	5958	6996	8062	9158	$\bar{0}285$					
83	8.1443	2636	3863	5126	6427	7769	9152	$\bar{0}579$	$\bar{2}052$	$\bar{3}572$					
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20					
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					

Difference-columns cease to be useful, owing to the rapidity with which the value of the tangent changes.

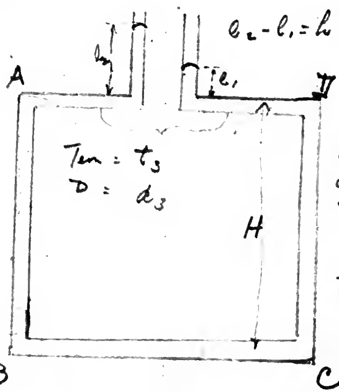


V_{90} for glass

$V_{90} = V_0\left[1+\alpha(9\right.$

$= 7.905(1+\alpha$

...25



$l_2 - l_1 = h$

Press at B = Pressure at C

$\cancel{p} + Hd_2g - l_2d_3g = \cancel{p} + Hd_1g - \ldots$

$\therefore (l_2 - l_1)d_3 = H(d_1 - d_2)$

or $hd_3 = H(d_1 - d_2)$

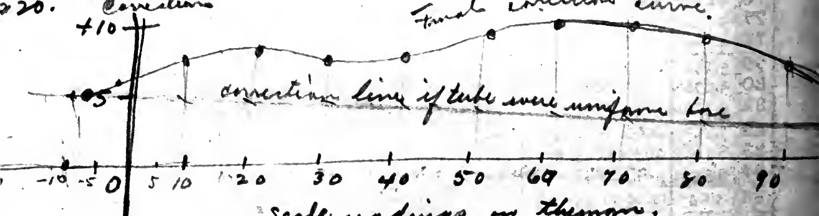
Cold
Tem t_1
dens d_1

$d_0 = d_1(1+\beta(t_1-0))$

$d_0 = d_2(1+\beta t_2 - 0)$

Fanced water $d_0 = d_3(1+\beta t_3 - 0)$

...20.



suppose that { In ice reading is $-5°$ true temp $0°$
{ In steam reading is $96°$ true temp $99.8°$ (...

$$\beta = \frac{d_1 - d_2}{d_2(t_2 - t_1)}$$

$$= \frac{\dfrac{m - m_2}{V_1} - \dfrac{\cancel{M} - m_3}{V_1\left[1 + \alpha(t_2 - t_1)\right]}}{\dfrac{M - m_2}{V_1\left(1 + \alpha(t_2 - t)\right)}\left((t_2 - t_1)\right)}$$

1808½ alert

189926

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